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BOOK REVIEWS

Traveling wave solutions to PDEs, especially spectral and dynamic stability of nonlinear waves, is the focus of a new Springer book by Kapitula and Promislow. Arnd Scheel knowingly writes about it in our featured review. Other reviews provide a reader's potpourri of topics ranging from mapping class groups to texts on PDEs, mathematical physics, numerical methods in finance, and numerical methods in general. You can also learn about object tracking, 2-D turbulence, and analytic perturbation theory. Our reviewers again provide their valuable recommendations to help you select what to read and consult.

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Book Reviews

Edited by Robert E. O'Malley, Jr.

Featured Review: Spectral and Dynamical Stability of Nonlinear Waves. By Todd Kapitula and Keith Promislow. Springer, New York, 2013. \$79.95. xiv+361 pp., hardcover. ISBN 978-1-4614-6994-0.

Undergraduate courses on partial differential equations (PDEs) rarely venture beyond the idea of inherently linear eigenfunction expansions or separation of variables techniques. A glimpse into the world of nonlinear PDEs is sometimes offered when studying special solutions, either explicit or of a particular form. Standing or traveling waves, such as pulses or fronts, can indeed be constructed in many examples using quite elementary techniques. Beyond direct numerical simulations, looking at simple one-dimensional setups and focusing on simple solutions of the form $u(x - ct)$ is often the only route to gain insight into the temporal dynamics. The traveling wave ansatz is commonly known as the “ODE reduction,” since it reduces the PDE to an ordinary differential equation (ODE) for which one then looks for bounded solutions, in particular, periodic, homoclinic, or heteroclinic orbits. The method is particularly successful if one can reduce the ODE to a planar system, where the Poincaré–Bendixson theorem helps to classify bounded orbits. Famous examples of such special traveling-wave solutions arise as solitary waves in dispersive equations, such as water wave problems or equations in nonlinear optics, or as pulses and fronts in reaction-diffusion-type systems. From the perspective of this book, one might mention solitary waves

$$u(t, x) = \frac{c}{2} \operatorname{sech}^2 \left(\frac{\sqrt{c}}{2} (x - ct) \right)$$

in the KdV equation

$$\partial_t u = -\partial_{xxx} u - 6u \partial_x u$$

and excitation pulses $(u, v)(x - c_* t)$, $c_* = \sqrt{2}(\frac{1}{2} - a) + O(\varepsilon) > 0$, in the FitzHugh–Nagumo system

$$\begin{aligned} \partial_t u &= \partial_{xx} u + u(1 - u)(u - a) - v, \\ \partial_t v &= \varepsilon(u - \gamma v). \end{aligned}$$

The first and quite relevant objection to the traveling-wave reduction approach is that such solutions correspond to very particular initial conditions, so they are almost never observed in experiments or simulations. The present book can be viewed in many ways as the natural response to this objection. In short, traveling waves are relevant and informative when open sets of initial conditions stay close to a traveling wave for long

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time intervals. Such information can be obtained by studying the linearization at a traveling-wave solution, in particular spectral properties. This book by Kapitula and Promislow provides a quite unique entry point into this area, suitable for graduate students and young researchers who are interested in entering the field.

The question of the relevance of traveling waves has a counterpart: why should one be at all surprised to see traveling waves, or, more generally, coherent behavior? In damped-driven systems, this question can be rephrased from a higher vantage point as a search for the emergence of self-organized behavior in far-from-equilibrium systems. Striking examples of such self-organized behavior are, for instance, spiral waves in chemical reactions. In conservative, Hamiltonian systems, Poincaré's recurrence theorem excludes asymptotic stability. Linearized or spectral stability still give good indications of stability when one can establish Nekhoroshev-type estimates, thus guaranteeing that solutions stay close to the equilibrium for exponentially long time spans. Stronger stability results exploit energetic stability or constrained stability. A different route, not taken here, would rely on dispersive estimates to show even stronger, asymptotic stability [10].

Many of the ideas around spectral, linear, and nonlinear stability became accessible to a larger audience through Dan Henry's lecture notes [8]. Inspired by dynamical systems ideas, spectral properties are viewed as universal criteria, independent of coordinate changes, when the goal is to characterize dynamics in the vicinity of equilibria. Henry's book is also one of the first references to consider characterizations of spectral properties of the linearization of traveling waves and, using center manifolds, the nonlinear stability of traveling waves. In fact, both spectral properties and nonlinear stability are complicated by the fact that traveling-wave problems are typically posed on the whole real line. The loss of compactness is reflected in the presence of essential spectra that need to be controlled and understood. The translation symmetry on the real line causes the presence of a neutral eigenvalue at the origin, so that the linear evolution does not cause exponential decay, but merely exponential decay to a constant.

The authors discuss both of these aspects carefully in Chapters 3 and 4 after setting up some basic preliminaries and notation from functional analysis in Chapter 2, which also includes a nice review of Sturm–Liouville theory. Chapter 3 contains, in particular, a characterization of essential spectra for operators with asymptotically constant coefficients (homoclinic and heteroclinic traveling waves) and for operators with periodic coefficients (periodic wave trains). It also explains in an informal way the effect of domain truncation, following [11], a phenomenon that is revisited in Chapter 10.

Chapter 5 covers orbital stability in Hamiltonian PDEs, motivated largely by the work of Grillakis, Shatah, and Strauss [6, 7]. In the absence of effective dissipation mechanisms, one can strive to show stability using the conservation of the Hamiltonian. In fact, if an equilibrium is a strict minimum of the Hamiltonian, nearby trajectories necessarily stay in a neighborhood, which roughly establishes Lyapunov stability of the equilibrium. In the case of traveling waves for Hamiltonian PDEs, equilibria come in families due to translation symmetry. Also, equilibria are typically not minimizers of the energy. On the other hand, translation symmetry corresponds to another conserved quantity by Noether's theorem, the momentum. It is therefore sufficient to show that the Hamiltonian possesses a strict minimum when restricted to a level set of the momentum. Such ideas can lead to proofs of stability of traveling waves in some contexts and, more generally, to counting arguments for unstable eigenvalues of the linearization. Both aspects are carefully covered in Chapters 5 and 7,

although the most general result, the Hamiltonian Krein index theorem, is stated but not proved in full generality. Chapter 7 also contains a very interesting collection of perturbation results, covering, for instance, symmetry-breaking effects and dissipative perturbations. Chapter 6 develops classical perturbation theory for eigenvalues in the point spectrum, a necessary prerequisite for various results in subsequent chapters.

Chapters 8 through 10 are concerned with the Evans function, which was introduced in the 1970s in a series of articles to study the stability of pulses in nerve axons [2, 3, 4, 5]. It gained traction later when it was used to prove stability of the FitzHugh–Nagumo pulse [9] and the KdV soliton [10]. The paper by Alexander, Gardner, and Jones [1] today is viewed as the basic (and first) reference for properties of the Evans function. While the Evans function is widely used, in hyperbolic conservation laws, nonlinear optics, water wave models, and in reaction-diffusion systems, there are few expository articles, let alone textbooks, from which beginning graduate students can learn the subject. In this respect, Chapters 8 through 10 are a very valuable asset for the community. The construction here is slightly more hands-on and explicit than in other review articles. Instead of relying on differential forms as in [1], the authors use specific Jost solutions to build the Wronskian for the Evans function.

Roughly speaking, the Evans function gives a reduction of an eigenvalue problem of the form $\mathcal{L}u = \lambda u$, with \mathcal{L} a k th-order differential operator on the real line or a bounded interval, to an analytic function $E(\lambda)$, so that roots of E coincide with eigenvalues of \mathcal{L} . The construction writes the eigenvalue problem as a first-order differential equation and uses the linear evolution of this differential equation to transport boundary conditions to the origin $x = 0$ as linear subspaces. One then tracks intersections of these subspaces by forming a determinant of basis vectors in these subspaces.

While a reduction of an eigenvalue problem to finding the roots of a single analytic function is obviously beautiful and striking, it is only the fact that this function (or at least some properties) is computable that turns the Evans function into a powerful tool in the study of stability of nonlinear waves. The authors illustrate this nicely, relying mostly on explicitly solvable scattering problems.

Another strength of the Evans function (and this book) is the possibility of tracking eigenvalues when they disappear into the essential spectrum. The simplest example is that of small localized potentials in Schrödinger operators,

$$\frac{d^2 u}{dx^2} + \varepsilon V(x)u = \lambda u, \quad V(x)e^{\eta|x|} \rightarrow 0 \text{ for } |x| \rightarrow \infty,$$

for some $\eta > 0$. The Evans function turns out to be an analytic function of $\gamma = \sqrt{\lambda}$ for all $\varepsilon \neq 0$ and all $\Re \gamma > -\eta$. For $\varepsilon = 0$, the Evans function $E(\gamma) = 2\gamma$ possesses a simple root at the origin. A perturbation calculation gives

$$E(\lambda) = 2\gamma - \varepsilon \int V + O(\varepsilon^2).$$

The eigenvalue $\lambda \sim (\varepsilon \int V/2)^2$ turns into a resonance pole when $\varepsilon \int V < 0$, with corresponding eigenfunction $u \sim e^{|\gamma x|}$. The book spends quite some time on carefully introducing the Riemann surface branch cuts more generally and provides us with more explicit, integrable examples. One of the highlights of the book is Chapter 10.4, where those results are applied to a perturbation calculation for eigenvalues located at the edges of the essential spectrum in integrable systems.

A somewhat simpler application of the Evans function is that of parity indices. Computing the sign of the Evans function for small $\lambda > 0$ and for $0 < \lambda \rightarrow \infty$, one

can deduce the parity of the number of positive eigenvalues. This has been exploited in numerous contexts to give instability criteria [12, section 6], but also to show stability [9]. Key to these results is the connection between the Evans function and its derivatives at $\lambda = 0$ and the geometry of stable and unstable manifolds in the construction of traveling waves. In the simplest case, the derivative of the Evans function is given by a Melnikov integral that measures the splitting distance between stable and unstable manifolds as the wave speed is varied as a parameter in the traveling-wave ODE.

The authors also consider the truncation problem, when pulses or fronts are considered on a large but finite interval in a comoving frame with suitable separated boundary conditions [11]. Spectra converge exponentially as the domain size increases, and the authors give a constructive proof based on exponential convergence of Jost solutions and Evans functions. An indirect argument is used to show a clustering result for eigenvalues at the absolute spectrum. Although this result is slightly weaker than the original convergence result in [11], it serves as an excellent stepping stone for those interested in diving deeper into the theory.

Overall, I found this book to be an extremely valuable asset for students and young researchers who want to get into this field. One can find endless lists of topics that should have been covered in such a book. Numerous prerequisites from functional analysis and classical PDE theory are covered only tangentially. Similarly, results from and connections with inverse scattering theory are only touched upon. However, it is a strength of the book that it has narrowed the scope. As is, the book does an excellent job at stimulating readers to get their hands dirty and play around with the examples offered. The book is, surprisingly, largely self-contained, and it proves most of the key results rigorously, sometimes restricting to the simplest case. An extensive bibliography and plenty of remarks at chapter endings then serve as a guide to history and current literature. This field has needed such a book as an entry point for graduate students, and the authors deserve a huge thanks from the community for putting it together.

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Mathematical Foundations of Imaging, Tomography and Wavefield Inversion. By *Anthony J. Devaney*. Cambridge University Press, Cambridge, UK, 2012. \$82.00. xviii+518 pp., hardcover. ISBN 978-0-521-11974-0.

Inverse problems and, in particular, imaging problems are a staple of applied mathematics, engineering, and physics, and many scientists earn their bread and butter addressing issues of medical imaging, sonar/radar-based remote sensing, nondestructive testing and evaluation, etc.. Since these applied problems are all grounded in wavefield inversion methods, it might be expected that a common interdisciplinary text appropriate to all students would have appeared long ago. But—perhaps because of the considerable prerequisite training required to understand the physics of radiation and scattering, the issues of ill-posed problems and regularization methods associated with the solution of integral equations, and the practical problems of constructing reliable computer algorithms—it has been hard to strike that Goldilocks balance between “too much of this” and “too little of that.” The more common result is a book written for a smaller audience consisting of mathematicians *or* physicists *or* engineers. Tony Devaney has now produced just such an interdisciplinary book addressed to all audiences.

Of course, this task requires both physics and mathematics preliminaries, and Devaney starts by introducing the fundamental tools for dealing with initial and boundary value problems for the scalar wave and Helmholtz equations. Roughly constituting the first one-third of the book, he includes a review of the the Sturm–Liouville problem, multipole expansions, radiation fields, dispersive media, and other preliminary concepts in both the time and frequency domains (this alone could be a valuable, if succinct, reference). Throughout he also

includes an extensive discussion of Green functions for the radiation problem, and this consolidated exposition is a wellspring for the uninitiated student. Devaney has staged these basic results up-front so that they can be referenced when needed later in the text and, owing to the interdisciplinary nature of the book, the layout makes good sense. While so much uninterrupted background material (four chapters) concentrated in one place might dishearten a student who is anxious to “get on with imaging,” the formal feel of these chapters is significantly lessened by the inclusion of very informative examples—many of which anticipate later results. To complement this necessarily compressed background overview, each chapter includes thoughtfully crafted exercises to amplify the main points of the exposition.

Chapter 5 begins the text’s subject in earnest with the inverse source problem (ISP). After developing the ISP integral equation, Devaney proceeds systematically to discuss solutions for a number of important sources and geometries. Along the way he introduces singular value decomposition (SVD) methods and discusses their implementation. Mindful to discuss the consequences of nonideal data, this chapter provides a wealth of valuable information, and I was especially glad to see the SVD-based exposition both here and continued through the remainder of the book (something that is not as common as it should be in such texts).

Chapter 6 is preliminary to the inverse scattering problem (ISCP) and introduces scattering and diffraction. After developing the Lippmann–Schwinger integral equation and discussing various general properties of the scattered field, Devaney moves on to the various schemes for approximating solutions (Born, Rytov, physical optics, etc.) and the inversion techniques based on these approximations in Chapters 7 and 8. Here,

as before, his approach is to illustrate ISCP methods by examining specific problems involving data constraints and scattering geometries.

The final three chapters deal with some of the odds and ends that have been deferred to this point so as to more expediently develop the main themes of the book. Chapters 9 and 10 address the very important problem of wavefield inversion in *inhomogeneous* media, while Chapter 11 addresses the *vector* Helmholtz equation associated with electromagnetic fields (a topic too often brushed aside in standard imaging texts). Starting with an overview of the Maxwell equations, Devaney develops electromagnetic radiation and scattering in an amazingly concise tour de force that describes the similarities as well as the differences between scalar and electromagnetic waves.

This is the kind of remarkable and comprehensive book that I would expect to be penned by one of the foremost experts in the field and Prof. Devaney is to be congratulated. As a reference source this book is certain to prove its value. As a text, however, the book (in its entirety) might be a bit challenging for a one-semester introductory graduate-level course offered to students who really do have only “rudimentary familiarity with the wave and Helmholtz equations in a homogeneous medium.” The reason, of course, is the sheer amount of material covered—the book intends to “present the mathematical (and physical) foundations of imaging,” and that is a tall order. A preliminary course in Fourier optics could help ease the passage into these more advanced topics. However, the book’s exposition allows it to be tailored to a course with more modest goals. Other than that, I believe the author has found a marvelous balance between physics and mathematics, and, even though he laments that this was accomplished “at the expense of mathematical rigor,” I think he got it just right.

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A Primer on Mapping Class Groups.
By Benson Farb and Dan Margalit. Princeton University Press, Princeton, NJ, 2012.

\$75.00. xiv+472 pp., hardcover. ISBN 978-0-691-14794-9.

The classification of surfaces (or, more formally, of closed two-dimensional manifolds) has been known since the 1860s and describes how they are all obtained from spheres by adding finitely many handles or crosscaps. Restricting to oriented surfaces gives an even simpler picture: surfaces are classified by a single integer—the *genus*—which can be any nonnegative integer; this integer can be computed easily from any combinatorial description of the surface (e.g., from the Euler characteristic χ , which is related to the genus g by the formula $\chi = 2 - 2g$). Thus, a closed oriented surface of genus 0 is a sphere, a closed oriented surface of genus 1 is a torus, and so on. However, as is so often the case in mathematics, the real interest lies not so much in the objects themselves, but in their groups of *symmetries*.

If S is a closed, oriented surface, the group $\text{Homeo}^+(S)$ of orientation-preserving self-homeomorphisms is extremely complicated as an abstract group. However, it may be topologized in an obvious way (i.e., with the compact-open topology) by how it acts on S , and then as a space it becomes much more manageable (at least up to homotopy). If we denote by $\text{Homeo}_0(S)$ the connected component of $\text{Homeo}^+(S)$ containing the identity, then the situation is as follows:

1. The group $\text{SO}(3, \mathbb{R})$ of rotations of Euclidean 3-space acts by isometries on the round sphere S^2 and defines an inclusion $\text{SO}(3, \mathbb{R}) \rightarrow \text{Homeo}_0(S^2)$, which is a homotopy equivalence.
2. The square torus $\mathbb{R}^2/\mathbb{Z}^2$ acts on itself by translations and defines an inclusion $\mathbb{R}^2/\mathbb{Z}^2 \rightarrow \text{Homeo}_0(T)$ which is a homotopy equivalence.
3. For S a surface of genus at least 2, $\text{Homeo}_0(S)$ is contractible.

One can already see from this a fundamental distinction in the study of surfaces: the case of genus 0 and 1 is exceptional, whereas all surfaces of genus at least 2 exhibit similar phenomena (for surfaces with punctures or boundary components there is a similar distinction, but in this case what is im-

portant is not the genus, but the sign of the Euler characteristic). Because the action of $\text{Homeo}^+(S)$ on itself is continuous, the group $\text{Homeo}_0(S)$ is a *normal* subgroup of $\text{Homeo}^+(S)$, and the quotient $\text{Mod}(S)$ is called the *mapping class group*. Thus—for a surface S of genus at least 2—understanding the group $\text{Homeo}^+(S)$ up to homotopy is tantamount to understanding the mapping class group of S .

Surfaces and their symmetries are ubiquitous throughout geometry and, even more broadly, throughout mathematics. In the first place, surfaces arise as *Riemann surfaces* and therefore can be found wherever one finds the complex numbers (which is to say, everywhere). Moreover, surfaces frequently arise in *families*, and the global study of these families is governed by mapping class groups. For example, a meromorphic function on a complex surface (which is topologically four-dimensional) has fibers which are (possibly singular) Riemann surfaces, and the monodromy of the nonsingular fibers is a subgroup of a mapping class group. More generally, any symplectic 4-manifold admits the structure of a *Lefschetz pencil*, whose nonsingular fibers are surfaces, and these can also be studied by mapping class groups. Many 3-manifolds have the structure of a *surface bundle* over a circle, so that they are described exactly up to homeomorphism by giving a single conjugacy class in some mapping class group. The dynamics of flows on 3-manifolds is captured by return maps to a two-dimensional cross-section, and many important features of flows are encoded in the relative *braiding* of orbits around each other; such braiding is parameterized by elements of *braid groups*, which are mapping class groups for certain punctured surfaces. So the importance of the theory of mapping class groups to many different mathematical constituencies is beyond question. What has been sorely missing is a clear, readable introduction to the subject that lays out the basic facts with a minimum of fuss and with an eye to their applications—in short, a primer.

The view of surfaces as geometric objects has a lot to recommend it, and one of the canonical routes to the understanding of mapping class groups is through the study of geometric structures on surfaces, espe-

cially holomorphic structures, and Riemannian metrics. Every surface admits a canonical smooth structure, and every smooth manifold admits a Riemannian metric. For an oriented surface, any conformal class of Riemannian metric determines a unique compatible holomorphic structure (i.e., a realization as a Riemann surface) and, furthermore, every conformal class of metric contains a unique representative of constant curvature -1 , 0 , or 1 up to the ambiguity of scale in the case of curvature 0 ; this is more or less a restatement of the Riemann mapping theorem. The sign of the curvature is the same as the sign of the Euler characteristic χ , and the distinction between phenomena associated with surfaces of genus at least 2 and surfaces of genus 0 or 1 can be explained by *geometry*. Thus one is quickly led to study the *moduli space* of metrics on S of constant curvature -1 (at least when the genus of S is bigger than 1). Here one obtains one of the most fruitful interpretations of mapping class groups: as (orbifold) fundamental groups of moduli spaces.

The geometry and topology of moduli spaces and their compactifications is very beautiful, but also very complicated, so it is natural to look for simpler spaces that capture some of the information, but are easier to study directly. Associated to a Riemann surface in a natural way (taking the quotient of $H^1(S; \mathbb{R})^*$ by the *period lattice*) is a certain kind of high-dimensional torus called a *Jacobian*. Families of Riemann surfaces give rise to families of Jacobians, and Jacobians are examples of what are called *principally polarized abelian varieties*, which have their own moduli space; this is much easier to understand than the moduli space of a surface. For example, the moduli space of principally polarized abelian varieties has (orbifold) fundamental group equal to the symplectic group $\text{Sp}(2g; \mathbb{Z})$, where g is the genus, so we obtain a natural “forgetful” map $\text{Mod}(S) \rightarrow \text{Sp}(2g; \mathbb{Z})$. This map between groups may be seen more easily just by thinking about the action of a mapping class on the one-dimensional cohomology $H^1(S; \mathbb{Z})$, which is naturally made into a lattice in a symplectic vector space by the cup product. Thus we can achieve insight into the algebraic structure of the mapping class group by studying both the symplec-

tic group $\mathrm{Sp}(2g; \mathbb{Z})$ and the kernel of the map $\mathrm{Mod}(S) \rightarrow \mathrm{Sp}(2g; \mathbb{Z})$, which is called the *Torelli group* \mathcal{I}_g . This latter group is very mysterious, but some of its structure is revealed by studying its homomorphisms to abelian groups, following the approach of Dennis Johnson.

Finally, a concern somewhat complementary to the study of mapping class groups themselves is the problem of finding a representative self-homeomorphism that is the best avatar of its mapping class or has properties that make it especially useful for applications or computation. Here one has the *Nielsen–Thurston classification theorem*, which says that each mapping class has a representative ϕ which is of one of the following types:

1. ϕ is *finite order*—i.e., $\phi^m = \mathrm{id}$ for some positive m ;
2. ϕ is *reducible*—i.e., it leaves invariant a finite collection of pairwise disjoint essential simple closed curves in S ; or
3. ϕ is *pseudo-Anosov*.

Finite-order automorphisms are very easy to understand. Reducible automorphisms restrict to automorphisms of simpler surfaces with boundary (obtained by cutting along the invariant system of curves). Therefore, the interesting automorphisms are those which have pseudo-Anosov representatives. The simplest way to define a pseudo-Anosov homeomorphism is to say that in local charts it looks like an affine map of the plane which stretches one axis by some factor $\lambda > 1$ and the other axis by a complementary factor $1/\lambda$, except that there are finitely many points where the dynamics looks more like a “branched cover” of such an affine map, of degree $n/2$ for some integer $n \geq 3$. A more rigorous definition is to say that there is a holomorphic quadratic differential ω on the underlying Riemann surface that defines singular foliations tangent to the directions in which a square root $\sqrt{\omega}$ is real or imaginary, and that ϕ stretches the leaves of the real foliation by λ and the leaves of the imaginary foliation by $1/\lambda$ (the singularities correspond to the zeros of ω). Pseudo-Anosov representatives are very nice to work with for many reasons; for instance, they admit nice Markov partitions, and they have the sim-

plest dynamics (as measured by entropy) of any representative of their mapping class.

The book of Farb and Margalit does an awesome job of covering all these different perspectives (and much more) and explaining the basic phenomena, examples, and applications clearly, unambiguously, and accessibly. The book is divided into three sections; we briefly discuss the contents of each section.

The first section is concerned with the algebraic structure of mapping class groups and their relation to the combinatorial topology of two-dimensional surfaces. The most important results in this section are the Dehn–Nielsen–Baer theorem that $\mathrm{Mod}(S)$ is isomorphic to the outer automorphism group of $\pi_1(S)$ (up to $\mathbb{Z}/2\mathbb{Z}$ parameterizing orientations) and the fact that $\mathrm{Mod}(S)$ is finitely presented. One of the highlights here is that the authors give several different finite (and infinite!) presentations and discuss the virtues and shortcomings of each one. This section also describes the relationship between $\mathrm{Mod}(S)$ and $\mathrm{Sp}(2g; \mathbb{Z})$, and it contains a discussion of the Torelli group and the Johnson homomorphism.

The second section is concerned with the geometry of moduli space and its universal (orbifold) cover Teichmüller space. Compared to the size of this subject, this section is comparatively brief; nevertheless, it contains a good introduction to the theory of quasi-conformal maps, Teichmüller’s theorem on the existence and uniqueness of maps minimizing the dilatation in a given homotopy class of homeomorphism between Riemann surfaces, and Mumford’s compactness criterion and the topology of the “end” of moduli space.

The third section is concerned with the Nielsen–Thurston classification of mapping classes, and especially with the structure of pseudo-Anosov elements. These can be understood in many different ways: with holomorphic quadratic differentials, measured singular foliations, transversely measured geodesic laminations, weighted train-tracks, and so on. Much of this approach is due to Thurston, and the book presents Thurston’s approach to the subject and sketches his proofs of key results.

The authors manage throughout to convey a coherent, comprehensive, and inte-

grated vision of the theory of surfaces and mapping class groups that explains how all the different ways of thinking about these objects fit together, and how they connect with the rest of mathematics. This book is thus timely and welcome, and should be of tremendous value to mathematicians of every stripe.

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Fundamentals of Object Tracking. By Subhash Challa, Mark R. Morelande, Darko Mušicki, and Robin J. Evans. Cambridge University Press, Cambridge, UK, 2011. \$95.00. xii+375 pp., hardcover. ISBN 978-0-521-87628-5.

This book deals with the important area of multiple object tracking or multitarget tracking. Advances in sensor technology have provided data sources for tracking moving objects in civilian applications such as air traffic control or video surveillance, and military applications such as intelligence, surveillance, and reconnaissance (ISR). The objects of interest may move underwater, on the surface or land, or in the air or space. Sensors may include radar, infrared, video, acoustic, seismic, etc. Tracking problems also exist in the cyber domain, where the objects are network activities instead of physical entities and sensors are intrusion detection devices on the communication networks and computers. Some examples are given in Chapter 1 of this book.

Despite the importance of object tracking, there are not many books that provide the fundamentals of object tracking for students or engineers who are new to the field. Until the 2011 book by Bar-Shalom, Willett, and Tian [1], the most recent book was [2], published in 1999. Readers who want to learn about object tracking have to read papers scattered in many journals and conference proceedings. Reference [1] is encyclopedic and includes many algorithms, but it is huge at over 1200 pages. Thus, there was a need for a more compact book that provides the basics of object tracking without burdening the reader with too many details. These authors have satisfied this need with a well-written book that starts with “the generic Bayesian solution”

and “then shows systematically how to formulate the major tracking problems . . . and how to derive the standard tracking solutions.” The systematic treatment includes explicit statements of assumptions such as point objects with dynamics described by a Markov process and point measurements from sensors with infinite resolution. Even though these assumptions are not valid for problems such as video surveillance, they are good approximations for many problems and necessary for developing implementable algorithms.

A unique feature of multiobject tracking is that the sensor measurements frequently do not come with the identities of the objects that generate them. Thus association of measurements to objects is needed before object states can be estimated. Data association is the main difference between object tracking and state estimation or filtering, where algorithms such as Kalman filtering assume known measurement origins. Traditional object tracking algorithms include two main components: state estimation of object tracks given associated measurements, and association of measurements to object tracks. The major part of this book discusses algorithms for these two components using a Bayesian approach.

The Bayesian approach is introduced in Chapter 1, which formulates object tracking as a general Bayesian estimation problem and presents recursive equations for prediction and update of conditional probability densities. These equations are only notational because the object states and measurements are random sets and not random vectors. In fact, the same equations appear later in Chapter 6: multiple object tracking in clutter, a random-set-based approach. Similar equations for general Bayesian multiple object tracking appear in [3], which should have been referenced if only because of its title, *Bayesian Multiple Target Tracking*.

Chapters 2 and 3 focus on the filtering component of object tracking and present algorithms for generating state estimates and associating measurements. Even though the chapter titles include the word “tracking,” the origins of the measurements are assumed known. Chapter 2 presents traditional algorithms such as the Kalman filter and the extended Kalman filter and

more recent nonlinear algorithms such as unscented filter, point mass filter, and particle filter. An example shows the excellent performance of particle filters, but a comparison of the computational costs would have been useful. Chapter 3 presents filters for maneuvering objects including the generalized pseudo-Bayesian filters, the interacting multiple model (IMM) filter, its variable structure variant, and the particle filter for maneuvering objects.

Chapters 4 and 5 address the data association problem in object tracking. Chapter 4 deals with measurement association to an existing object track, e.g., whether the measurement is from the object or clutter. It presents standard approaches such as nearest neighbor filter, probabilistic data association filter (PDAF), and particle filter in clutter. Chapter 5 is the longest chapter (almost one quarter of the book) and addresses single and multiple object tracking using the object existence paradigm. This paradigm was developed by the authors to assess the quality of a track that is updated using the PDAF and extended to handle more complex problems. For nondeterministic object existence, when the number of objects is not known, joint integrated probabilistic data association (JIPDA) deals with single-scan association for multiple objects, and joint integrated track splitting (JITS) treats multiscan association for multiple objects. Single object algorithms just drop “joint” (J) from the names, and multiple model algorithms add IMM before the names.

The JITS algorithm for multiobject multiscan tracking is similar to multiple hypothesis tracking (MHT). Even though JITS does not use the standard MHT terminology of tracks and hypotheses, it also generates new branches of a track tree at each scan to account for different association alternatives and computes the association probabilities recursively. Since the number of alternatives (called track components in the book) grows rapidly, practical implementation also requires standard MHT control techniques such as merging and pruning. It is understandable that the authors focus more on their own algorithms in their book, but more discussion of MHT and reference to classical papers such as [4] would have been useful because MHT

is a more popular approach than JITS for multiobject tracking.

Chapter 6 takes readers back to the recursive Bayesian approach introduced in Chapter 1 by discussing the random set approach [5] that does not require separation into filtering and association. The last three chapters of the book deal with Bayesian smoothing using the augmented state approach, tracking with time-delayed and out-of-sequence measurements, and some practical issues in tracking. Three appendices provide the necessary mathematical background.

The authors have organized the content with a very readable format. Most chapters follow the same structure of problem statement, derivation, algorithms, performance bound, and illustrative example. All 43 algorithms are summarized in tables that provide the necessary equations for implementation. Most chapters end with an example that shows the implementation of the algorithms and compares algorithm performance. More examples would be desirable, especially for the unconventional applications discussed in Chapter 1, but this would be difficult given the small size of the book, which also forces the authors to skip important issues such as tracking with multiple sensors, extended objects, unresolved measurements, etc.

In summary, this book achieves its objective of providing the fundamentals of object tracking. By starting with the Bayesian approach, it systematically presents the most important algorithms given some fairly general assumptions, and it does this in a small package. One might wish that it had included more discussions of multiobject multiscan algorithms besides those developed by the authors; however, it contains almost everything most readers need to learn about the fundamentals of object tracking.

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Introduction to Partial Differential Equations. By Peter J. Olver. Springer International Publishing, Switzerland, 2014. \$69.99. xxvi+635 pp., hardcover. ISBN 978-3-319-02098-3.

There are plenty of good textbooks for a first course on partial differential equations (PDEs) suitable for one-semester courses or for full-year courses. The classic and widely adopted text by Haberman [6] comes to mind, as do texts by (in subjective order of increasing sophistication; for the record, I like all of these for many, sometimes very different reasons) Farlow [4], DuChateau [3], Asmar [1], Weinberger [11], and Strauss [9]. Why should one add another to the list? The answer is obvious, although probably provocative: because this new addition is the best one yet. In the interest of full disclosure, I should mention that I have taught from the notes that preceded this book, and I am mentioned in the acknowledgments because of this. In fact, it is useful to describe the level of the course I taught. The course had two sections: One section contained undergraduate students, mostly seniors, largely majoring in applied mathematics, mathematics, or physics. The other section, somewhat bigger, was aimed at graduate students from a large variety of science departments (atmospheric science, geosciences, oceanography, various engineering departments, etc.). The course lectures I prepared were identical: my level of presentation was somewhat more sophisticated for the graduate section, which received a few additional harder homework problems. Such a course is not atypical of those offered in many departments. What do I want from a textbook when I teach

this course? Here is an incomplete list: (a) I want to be able to cover all of the material required by the standard first-course syllabus: separation of variables, heat equation, Laplace equation, wave equation, and so on; (b) the text should point to the numerous connections to the application fields where much of this material originated; (c) the writing should be clear and understandable; (d) there should be enough material to entice students to go further, by taking another more advanced course, for instance; and (e) the book should have a wealth of homework problems.

This book easily covers all the material one might want in a course aimed at first-time students of PDEs. The three main components of a first course (heat equation, wave equation, Laplace equation) are present, and their various standard solvable boundary value problems are discussed. There are several very nice chapters on problems in multiple dimensions. All classical solution techniques are introduced: separation of variables leading to Fourier series, Fourier transforms, Green's functions, and d'Alembert's solution. One classical method that is missing is the Laplace transform. I am fine with this; I was never a big fan, but others may miss it. If so, [6] and others can be used to supplement this material. There are many references to the physical origins of the problems treated, although the approach is less grounded in applications from physics or engineering compared to, say, [6]. I should mention that some delightful modern applications are included such as the Black-Scholes equation from mathematical finance, image processing and denoising, and some quantum mechanics. It has always surprised me how underrepresented the latter is in the standard PDE texts. There are many smaller sections on related and more advanced mathematical topics inviting both students and teachers to go beyond what is available here: two chapters on numerical methods (see below) are an obvious example, but there are also glimpses of symmetry methods (did you see who wrote this book?) and nonlinear equations in short sections on the Burgers equation and the Korteweg-de Vries equation.

This book does not follow the approach of the typical first course in PDEs from which

the average student walks away thinking that they know how to solve PDEs: it might take some work, but it is doable. In fact, the typical first course in ODEs often has the same effect. This is the main reason I chose to teach from this book (or the notes, before the book was available) rather than any other. It presents the best first course on solution methods for PDEs, while admitting that we cannot solve most PDEs analytically. This admission is made in a few different ways. First, there is quite a bit of material on numerical methods for PDEs, in chapters on finite difference methods and finite elements. The latter chapter also discusses weak solutions. This fits in with the second approach, where a slow transition to qualitative methods is made through function spaces, adjoints, and maximum principles. This material allows for an easy transition to a more advanced course which in many other cases can be quite disjoint from a first course. The book strikes a near-perfect balance between rigor and accessibility, given the audience it is aimed at.

The book places less focus on complex analysis than a classic like [11] does. Not many newer books do, and this is not surprising. I am unhappy with this trend, as I have yet to see a complex analysis technique I dislike. I am not alone in this [7, 10]. I believe that a recent method due to Fokas [2, 5] will be part of the standard first-course PDE curriculum within a decade and it should find itself in textbooks soon. Since it requires a bit of complex analysis, perhaps the trend will reverse.

Let me discuss the chapters on numerical methods a bit more. Both are well done and fit in well with the other chapters. I believe students truly benefit from the treatment given here, within the context of PDEs. If you are teaching a one-quarter or one-semester course, it is unlikely your syllabus will have room for the inclusion of these chapters. On the other hand, if you have an entire academic year at your disposal, incorporating these chapters into your syllabus is a great idea.

The book is written very well. It is clear that the author is serious about conveying the message transparently, and it is equally clear that the text has gone through a number of iterations, resulting in very few if any

errors, typographical or otherwise. I recall teaching from the precursor notes two years ago and finding very few errors at that point, having covered at least parts of 8 of 12 chapters. The text has many homework problems, which range from run-of-the-mill boundary value problems, necessary so students can repeat steps shown in class without much change, to harder and usually more interesting ones. I found myself going outside the homework problems available in the notes on only a few occasions. The book is published by Springer, listed at \$70 (hardcover) or \$50 (e-book). That price is quite a bit lower than [6] or [9]; it is comparable to [1] (paperback edition), but more expensive than [3, 4, 11], all of which are Dover paperbacks. The hardcover version seems to be of good quality, although the cover has started to separate from the book on both of my copies. Some may be interested to know that an instructor solution manual containing detailed solutions to about half of the problems will be available in the near future.

In addition to other higher-level texts, Peter Olver has now given us a second undergraduate textbook, following [8]. Like the latter linear algebra textbook, I recommend this one highly: It provides the best first-course introduction to a vast and evermore relevant and active area. Students, and perhaps instructors too, will learn much from it. If they wish to go beyond the material taught in a first course, this text will prepare them better than any other I know.

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Classical and Multilinear Harmonic Analysis. Vols. I and II. By Camil Muscalu and Wilhelm Schlag. Cambridge University Press, Cambridge, UK, 2013. \$135.00. 760 pp., hardcover. ISBN 978-1-107-03262-0.

This two-volume set is a noteworthy addition to the expository literature on harmonic analysis. The authors have assembled a large amount of important material into a compact and well-organized package, and their exposition is terse but clear and well motivated.

The first volume offers a unified treatment of a number of important topics in classical harmonic analysis, beginning with Fourier series and leading up to more recent results such as Fourier restriction theorems and the $T(1)$ theorem, with emphasis on techniques such as Calderón–Zygmund theory and Littlewood–Paley theory that continue to play an important role in research. It provides an excellent resource for

readers who are familiar with the basics of Fourier analysis and wish to acquire a deeper understanding of the subject. The second volume deals with multilinear harmonic analysis, with emphasis on work of the first author and his collaborators. The central theme is the notion of paraproduct, and the high points include proofs of the by-now classic theorems on Calderón commutators, Cauchy integrals on Lipschitz curves, and almost everywhere convergence of Fourier series.

The reviewer has written a more detailed review of these books for *Mathematical Reviews* (review MR3052498 on MathSciNet).

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Mathematics of Two-Dimensional Turbulence. By Sergei Kuksin and Armen Shirikyan. Cambridge University Press, Cambridge, UK, 2012. \$85.00. xvi+320 pp., hardcover. ISBN 978-1-107-02282-9.

Two-dimensional turbulence has been a very active and intriguing area of research over the last five decades, since the publication of Robert Kraichnan’s seminal paper [1] postulating the dual cascade theory. Some reviews are given in [2, 3, 4]. The original motivation for studying two-dimensional turbulence was the belief that it would prove to be an easier problem than three-dimensional turbulence and that mathematical techniques developed for the two-dimensional problem would then be used for the three-dimensional problem. It was also believed that two-dimensional turbulence theory could explain flows in very thin domains, such as the large-scale phenomenology of turbulence in planetary atmospheres.

In general terms, theoretical studies of turbulence use a wide range of strategies, including phenomenological theories, analytic theories that depend on hypotheses established experimentally or via numerical simulations, and mathematically rigorous theorems on the Navier–Stokes equations. With a phenomenological approach, one makes a series of hypotheses based on experimental evidence and physical intuition from which conclusions can be drawn about

universal features of turbulence. The Kolmogorov theory of three-dimensional turbulence [5, 6, 7] and Kraichnan's theory of two-dimensional turbulence [1] typify this approach. In both cases, minimal contact is made with the Navier–Stokes equations. Nevertheless, a lot of successful numerical and experimental work has been motivated by phenomenological theories. With the more rigorous strategy of formulating analytical theories of turbulence, one uses the governing equations as a point of departure to formulate perturbative closure models or nonperturbative strategies. The mathematical foundation for the most advanced of these theories is the Martin–Siggia–Rose formalism [8, 9] (hereafter MSR formalism), with reviews given in [10, 11]. These theories cannot be completely rigorous on their own since the use of the MSR formalism entails certain assumptions: (a) existence and uniqueness of a deterministic solution for the velocity field given a choice of deterministic forcing field; (b) the assumption that the system was initialized at time $t \rightarrow -\infty$ and has already converged to statistical steady state. Furthermore, some lack of rigor stems from the dependence on Feynman path integrals. Finally, to connect theoretical predictions about ensemble averages with numerical simulations and experiments requires the additional assumption of ergodicity. Having made these assumptions, the payoff is that it is possible to make considerable inroads toward clarifying, explaining, and predicting the phenomenological behavior of turbulence for the two-dimensional as well as the three-dimensional case. Finally, another strategy is to prove mathematically rigorous theorems about the Navier–Stokes equations using functional analysis and dynamical systems theory techniques. This approach, pioneered by distinguished mathematicians like Leray, Foias, Temam, and many others, has successfully yielded solid results. The price is that it is too difficult to venture as far as one can go using less rigorous strategies that incorporate hypotheses evidenced by experiment or numerical simulations.

Ultimately, all of the above strategies have strengths and weaknesses that complement one another. A curious irony of two-dimensional turbulence research is

that whereas the phenomenology of two-dimensional turbulence is richer and poses many more challenges than that of three-dimensional turbulence, two-dimensional turbulence has turned out to be far more amenable to the pure mathematician's toolbox. The current book under review by Kuksin and Shirikyan surveys recent developments in the mathematical theory of the two-dimensional Navier–Stokes equations that are, with no exaggeration, quite breathtaking. The authors use the randomly forced two-dimensional Navier–Stokes equation with a regular Laplacian dissipation term at small scales as their ansatz. Three types of random forcing are considered: (a) kick forcing, consisting of, equispaced in time, delta function spikes with random amplitudes; (b) white noise, i.e., random Gaussian delta-correlated in time forcing, commonly used in MSR theories; (c) compound Poisson processes, which are random kick forces where both the amplitude and the temporal separation between the delta function spikes are randomized.

The authors begin in Chapter 1 with a very terse yet comprehensive review of essential concepts, needed for the proofs of the main results, from the areas of function spaces, measure theory, and Markov random dynamical systems. A solid graduate education in functional analysis is necessary to follow the chapter, but the authors provide citations to many other books that explain underlying concepts in more detail. Chapter 2 begins with a review of the classical Leray results on the existence, uniqueness, and regularity of solutions for the case of the two-dimensional Navier–Stokes equations with deterministic forcing. For the case of stochastic forcing, a series of important general results are proved that culminate in proving the existence of at least one stationary measure. In physical terms, a stationary measure describes the steady-state solution to the randomly forced Navier–Stokes equations when a dynamical balance has been established between forcing and dissipation and the ensemble averages for all observables become constant with respect to time.

The argument continues in Chapter 3 with an array of results establishing the

uniqueness of the stationary measure as well as the property of exponential mixing, both for periodic flows on an infinite domain and for flows on a bounded domain for various random forcing configurations. In physical terms, the property of exponential mixing means that regardless of the initial condition, the randomly forced two-dimensional Navier–Stokes equation will statistically converge to the steady-state solution at an exponential rate. This convergence has been established for both the velocity field itself and relevant observables, dependent on the velocity field, such as the energy spectrum. The authors also establish that if the random force is homogeneous, then the velocity field at steady state will also be homogeneous. The chapter concludes with a literature review as well as a physical summary of the main results.

Chapter 4 establishes ergodic theorems as well as some interesting limiting theorems. In particular, the authors establish that the time average of observables, dependent on the velocity field, quickly converge to the ensemble average as one extends the time interval over which the time average is taken. The authors also establish a central limit theorem that shows that the velocity probability distribution is close to Gaussian, in agreement with experiments and numerical simulations (see [3] for a review). Furthermore, the authors prove that the statistical properties of the velocity field at steady state will vary continuously as one varies the statistical parameters of random forcing. Finally, the authors prove that the steady-state solution of a system forced by random kicks will converge to the steady state of the system forced by white noise if the time gap between kicks is shrunk by a factor ε , taking the limit $\varepsilon \rightarrow 0^+$, as long as the amplitude of the kicks is also decreased by a factor of $\sqrt{\varepsilon}$.

Having established the existence and uniqueness of a stationary measure for the case of finite viscosity, in Chapter 5, the authors investigate the stationary measure under the limit $\nu \rightarrow 0^+$ of viscosity approaching zero. For technical reasons, instead of using a continuous limit it is necessary to work with the discrete limit $\nu_k \rightarrow 0^+$ with $k \in \mathbb{N}$ for some chosen viscosity sequence that converges to zero. The authors prove

that for every such viscosity sequence, the corresponding stationary measures have a nontrivial limit, as long as forcing is moderated by an $\sqrt{\nu_k}$ factor. It remains an open question whether all possible sequences such that $\nu_k \rightarrow 0^+$ with $k \in \mathbb{N}$ lead to a unique limit for the stationary measure. However, it is proved that all stationary measures obtained from any viscosity sequence limit to zero will satisfy certain universal properties from which a phenomenology of two-dimensional turbulence can be deduced. From these universal properties, if we introduce the assumption that the energy spectrum follows a power law, downscale from the forcing range, it is predicted that the energy spectrum will scale as k^{-a} with $a \geq 5$, where k is the wavenumber. The authors also identify an unproven conjecture that would rigorously imply $a = 5$.

Finally, in Chapter 6 the authors outline without proof a number of incomplete results whose development is the subject of current active research. A special highlight is a result that shows that the stationary measure of the three-dimensional Navier–Stokes equations, defined in a quasi-two-dimensional domain in which the vertical direction is very thin, and also randomly forced by random kicks, will converge to the corresponding stationary measure of the two-dimensional Navier–Stokes equations. However, it remains an open question whether this result can be extended for the case of white noise forcing.

In light of the foregoing discussion, the significance of these results is clear. In every well-known theory of two-dimensional and three-dimensional turbulence, one takes for granted the existence and uniqueness of the statistical steady-state solution, that a forced dissipative system will always converge to the steady-state solution, that the ensemble average can be exchanged with a time average, and that the discrete kick forcing typically used by numerical simulations, where time is discretized, properly approximates the case of continuous white noise forcing. These are all assumptions that underlie every theoretical effort to understand the phenomenology of turbulence, but they are also assumptions that are not easy to prove. It is very reassuring to see that during the last decade, at least for the case of

two-dimensional Navier–Stokes turbulence, all of these assumptions have been proved rigorously. This is a major achievement, and the authors are leading experts who have played a key role in the development of many of these results.

It is also worth commenting on the phenomenology of the k^{-5} energy spectrum predicted in Chapter 5. This is not an entirely new result. It was first proposed by Tran and Shepherd [12] and Tran and Bowman [13], who predicted a k^{-5} spectrum downscale from the forcing range and a k^{-3} spectrum upscale from the forcing range. This phenomenology is inconsistent with Kraichnan’s theory [1] of a downscale enstrophy cascade with k^{-3} scaling and an upscale inverse energy cascade with $k^{-5/3}$ scaling. As was explained by Tran and Shepherd [12], the Kraichnan cascades will fail to materialize in the absence of a dissipation term at large scales in a bounded domain flow. On an infinite domain, energy can simply cascade forever to larger and larger scales, and enstrophy can cascade to small scales and be dissipated by the small-scale diffusion term. However, on a finite domain, if there is no mechanism to dissipate the upscale energy cascade before it hits the largest possible scales, then the cascade configuration will collapse and transition to the conjectured joint k^{-3} and k^{-5} configuration. The results by the authors vindicate the work of Tran et al. [12, 13, 14] by eliminating unproven assumptions that they made in order to establish their predictions. As important as this development is, the greater challenge of understanding the robustness of the Kraichnan cascades remains an open question.

Finally, I should like to make some comments about the book itself. It has been written primarily for an audience of pure mathematicians who wish to familiarize themselves with this research area so they can make further contributions. The writing style is very concise; however, the authors provide complete proofs for almost all of their results. An extensive array of very general preliminary results needs to be established before the main results can be proved. The preliminary results are useful in and of themselves and can be used for the future investigation of systems other

than the randomly forced two-dimensional Navier–Stokes equations. The authors mention the complex Ginzburg–Landau equation as a possible area of exploration. An extensive bibliography of more than 200 references is given, and I very much appreciate the reverse citation system in which for each item in the bibliography the authors give the page numbers where the given item is cited in the text. Much heavy notation is used throughout the book; however, the authors provide a very useful summary of notation conventions at the end. Last but not least, in Chapters 3, 4, and 5 where the main results are discussed, the authors conclude each chapter with a very clear discussion of the physical implications of their results. These sections are essential to making this work accessible to a more applied audience. A very detailed literature review is also given at the end of every chapter for those who wish to consult the original research papers.

In summary, this is an excellent book presenting and proving a body of results that are of fundamental importance in the development of theories of two-dimensional turbulence. For pure mathematicians, there is much to be learned from the techniques used to prove the theorems that can be applied to a wider range of problems. For applied mathematicians, it is certainly useful to have some understanding of what has been proved rigorously for two-dimensional Navier–Stokes. The results themselves are very interesting and their physical implications are clearly explained. While this is not a book for the faint of heart, I find it an excellent addition to my library and strongly recommend it to everyone engaged in theoretical research on two-dimensional turbulence.

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Mathematical Physics: A Modern Introduction to Its Foundations. Second Edition. By Sadri Hassani. Springer International Publishing, Switzerland, 2013. \$99.00. xxxii+1205 pp., hardcover. ISBN 978-3-319-01194-3.

Mathematical Physics: A Modern Introduction to Its Foundations is intended for adoption in a course or a sequence of “methods of mathematical physics” at the advanced undergraduate or beginning graduate level. In this respect it will compete with other standard “methods” textbooks such as those of Boas [4], Arfken and Weber [2], Mathews and Walker [11], Byron and Fuller [5], and others.

The textbook has been reviewed positively by *Pure and Applied Geophysics*, *Zentralblatt Math*, and the European Mathematical Society newsletter, and excerpts appear on its Amazon and Springer web pages.

The title may be somewhat misleading. What are considered to be foundations by mathematicians may be quite different from the corresponding perception of physicists. The book does not discuss mathematical foundations in the spirit of Elliot Lieb's *Analysis* [10], for example. Instead, the text is written in a formal (but not abstract) way that emphasizes a general formulation of a topic before it is illustrated with numerous examples, usually drawn from the undergraduate physics curriculum. In this sense it differs significantly from the aforementioned “methods” texts, which are more informally written. Thus, the student of physics will be exposed to a necessary formalism whose applications are met repeatedly in the study of physics.

Particularly nice is Chapter 4, where the author addresses the algebra of endomorphisms of a vector space. The author defines polynomial operators $p(\mathbf{T})$ such as rotation operators, the exponential operator $\exp(\mathbf{T})$, commutators, projectors, and their calculus, all of which are used in a course of quantum mechanics, at the level of Sakurai [13]. Matrix representations of the operators are developed in Chapter 5 along with diagonalization and change of basis. The Dirac bra-ket notation is used throughout. Orthogonal polynomials are developed as vectors in Hilbert space rather than as

series solutions of second-order differential equations with singular points. The author develops the main points and theorems of infinite-dimensional vector spaces without hiding their essence behind puristic clutter. Chapter 6 discusses spectral decompositions and projection operators and has a very nice discussion of simultaneous diagonalization, which, for example, can be used classically in small oscillations or quantum mechanically for simultaneous measurements. A very nice discussion of the polar decomposition theorem follows, familiar to applied mathematicians from basic continuum mechanics, but here the author makes a connection with numbers and quantum mechanical operators. Basic information about Hilbert space and the Dirac delta function appear in Chapter 7 as well as a short discussion of distributions. Part IV discusses differential equations including existence theorems, power series solutions with applications to the quantum harmonic oscillator, and the WKB method. Part V discusses operators in Hilbert space with Sturm–Liouville systems as a special case, where the author develops the method of separation of variables.

I particularly liked Part VI, which is devoted to Green's functions, i.e., a topic that is scarce in the mathematics curricula unless you have the luck to be taught by an exceptional academic like Werner Weighlhofer, my Green's function teacher. Again, the author first builds the necessary formal background before illustrating the main points with examples drawn from classical electrodynamics and quantum mechanics. Part VII discusses basic group theory with emphasis on topics of interest in high energy physics such as group characters and tensor products of representations. Part VIII builds the formalism of tensor calculus, forms, symplectic geometry, and Clifford algebras. The forms are somewhat close in character to that of Flanders [6].

Part IX discusses Lie groups, algebras, and their representations. Two chapters of this part deserve special mention. Chapter 32 discusses symmetries of differential equations in the spirit of Peter Olver [12]. This important topic is written in an approachable manner that makes connections with standard quantum mechanical observ-

ables, e.g., angular momentum. Chapter 33 discusses calculus of variations in a very well written manner including the notion of functional derivatives that other competing texts prefer to avoid. In some sense its exposition is somewhat more rigorous than that of Gelfand and Fomin [7], but also more restricted in scope. In section 33.1.4 the author discusses the ideas of divergence and null Lagrangians on which one of the most important recent advances in existence theorems of nonlinear elasticity by John Ball and coworkers [3] is based.

Part X discusses fiber bundles, gauge theories, and Riemannian geometry. This last topic is very close to the exposition of Stewart [14]. Not only students of relativity, but also those of condensed matter field theory who read, for example, [1] and [9] will benefit, obtaining all required geometry foundations.

What are missing from this textbook are methods met in equilibrium and nonequilibrium statistical mechanics written in a rigorous but clear manner. I have found no such exposition in any textbook. Basic probability and stochastic processes at the level of Papoulis and Ross (models of probability) would be advantageous in the text. Also, perturbation theory is included in the Green's function sections but it would be more pedagogically correct if elementary ideas such as its application to singularly perturbed nonlinear oscillators could be discussed earlier, for example, in the differential equations part. In addition, some elementary results of perturbation theory and their associated diagrams might also be beneficial for the reader. The text would be more adaptable for self-study if the number of references was increased and citations appeared more frequently.

To conclude, this is a textbook that every library must have (with a large number of copies in reserve), and it will be a valuable aid for committed students, researchers, and instructors of science and applied mathematics. It is sold at almost half the price of some popular hardback differential equations textbooks offered by some mainstream publishers at extortionate prices. This is to the merit of Springer, and of the author for choosing a (relatively) low-cost publisher.

A few words about the author of *Mathematical Physics: A Modern Introduction to Its Foundations*. Sadri Hassani received his Ph.D. from Princeton University in high energy physics and is currently Professor of Physics at Illinois State University. He has strong and adamant views about the state of education in the United States. Being a skeptical educator, his views can be read in his personal blog at www.skepticaleducator.org.

Ralph Boas [8], reviewing *The Mathematics of Physics and Chemistry* by Margenau and Murphy, characterized its content as “pidgin mathematics,” i.e., “a clumsy and inept parody of mathematics” that can “be read by mathematicians who want to acquire a smattering of physics to impress their friends.” From Ralph’s standpoint, Hassani’s textbook also falls into this category. However, V. I. Arnold in his address in Palais de Découverte in Paris on March 7, 1997, stated that “the return of mathematical teaching at all levels from the scholastic chatter to presenting the important domain of natural science is an especially hot problem.” Hassani’s exposition does exactly that.

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Numerical Methods in Finance with C++.
 By Maciej J. Capiński and Tomasz Zastawniak.
 Cambridge University Press, Cambridge, UK,
 2012. \$39.99. x+166 pp., softcover. ISBN 978-0-521-17716-0.

Numerical Methods in Finance with C++ is part of the series “Mastering Mathematical Finance” and is an introduction to C++ programming targeted primarily at students in quantitative and mathematical finance masters programs. This book relies on the theory of option pricing to develop a sequence of programming tasks designed to introduce the reader to the C++ language, as well as to nonlinear solvers, Monte Carlo methods, and finite difference methods. The tasks in the first two (and, to some degree, the third) chapters are a particularly well-thought-out introduction to the C++ language and could easily form the core part of a syllabus for a financial programming course. The tasks in the numerical methods portion of the text serve to introduce the reader to several finance problems that lend

themselves to numerical solution, and also to reinforce earlier C++ concepts.

The authors take the somewhat novel approach of placing a code solution near (or, in one case, at) the beginning of each programming task, then following it up with a line-by-line description of what that code is doing. By using this design, they are able to introduce only the C++ language features and tools needed to tackle the task at hand, which prevents the reader from being overwhelmed by a comprehensive list of C++ capabilities. On the other hand, it does mean that some features are left out (e.g., protected members). However, these omissions are not too big of a drawback since, as the authors point out, there are many existing C++ manuals and online resources that the interested reader may turn to.

The book is organized as follows. The first three chapters comprise the introduction to the C++ portion of the book. An option pricer based on the binomial model for asset prices and the Cox–Ross–Rubinstein procedure is employed to introduce C++ language features: classes and subclasses, inheritance, virtual functions, multiple inheritance, and class templates. Chapter 4 uses nonlinear solvers (bisection and Newton–Raphson) to solve the implied volatility problem. Chapter 5 uses Monte Carlo methods to price path-dependent options. Finally, Chapter 6 uses finite difference methods to solve parabolic partial differential equations with the goal of pricing an American option.

This book falls roughly in the middle of the “Mastering Mathematical Finance” series and therefore assumes that the reader is already familiar with the underlying financial topics (in particular, option pricing). Further, while the book does start from scratch with respect to C++ (the author’s `Hi there` version of the ubiquitous `Hello world` program appears on page 3), readers who are already able to program (in some language) will likely have a much easier time working through the programming tasks.

My only criticism of this book is the lack of mention of a C++ compiler or integrated development environment (IDE). An IDE is such an integral part of C++ development that its use should be encouraged from the beginning. In particular, an IDE

makes debugging much easier. Presently, readers self-studying this book must deal with the significant additional step of figuring out how to compile their first program.

This book could be used as the primary text for a C++ programming course in a quantitative or computational finance program. The text is well organized and easy to follow, and the exercises are at the appropriate level. Proficiency with this subject matter should be viewed as prerequisite for a career in quantitative finance and/or programming.

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Analytic Perturbation Theory and Its Applications. By *Konstantin E. Avrachenkov, Jerzy A. Filar, and Phil G. Howlett*. SIAM, Philadelphia, 2014. \$89.00. x+369 pp., hardcover. ISBN 978-1-611973-13-6.

Konstantin Avrachenkov received a Ph.D. in 1999 from the University of South Australia, with Jerzy Filar and Phil Howlett as advisors. Since that time, the authors have further developed and applied the subject of analytic perturbation theory, together with a variety of coauthors, in both finite- and infinite-dimensional contexts. The end result is an authoritative SIAM monograph, intended as a textbook, filled with outstanding exposition and explicitly detailed examples.

The classical topic, linear and polynomial systems of algebraic equations depending analytically on a small parameter, is an important and challenging subject. Singular perturbations occur when the solution involves a Laurent expansion. (Thus, the setting is far removed from the usual differential equations formulations.) The focus here is on generalized inverses, Puiseux series, Groebner bases, the Newton polygon, mathematical programming, and Markov decision processes, in lovely series and matrix equations. Who would guess that the Google PageRank can be efficiently described as a singularly perturbed Markov chain?

Such problems become more varied as linear operators in Hilbert and Banach spaces,

where some functional analysis must be introduced. The authors clearly explain and motivate their development, leaving problems at chapter ends as well as bibliographic notes. If your interests include linear analysis, complex variables, optimization, and their applications, you'll find much of practical value here, eloquently presented.

Readers will find that there's much new to learn since Kato and Vishik and Lyusternik appeared in the 1960s!

ROBERT E. O'MALLEY, JR.
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Beautiful Geometry. By E. Maor and E. Jost. Princeton University Press, Princeton, NJ, 2014. \$27.95. xii+187 pp., hardcover. ISBN 978-0-691-15099-4.

This is a great book to buy your high-school age child (or grandchild) or their math teacher. It consists of 51 short chapters and an appendix with short proofs. The unusual topics vary from various geometry problems (with many references to Euclid and Coxeter) to numbers, symmetry, and the Reuleaux and Sierpinski triangles. Each chapter is well written and illustrated using detailed figures by the math historian/writer Eli Maor and is enhanced by colorful related plates (not necessarily mathematical) by the Swiss artist Eugen Jost. Compass constructions inspired, for example, "Seven Circles a Flower Maketh," while the number eleven suggests a Celtic motif. No college math is required, but thinking is.

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Five Decades of Tackling Models for Stiff Fluid Dynamics Problems: A Scientific Autobiography. By R. Zh. Zeytounian. Springer, New York, 2014. \$129.00. xxvi+153 pp., hardcover. ISBN 978-3-642-39540-6.

Radyadour Zeytounian was born in Paris in 1928, became an apprentice tailor, and emigrated to Armenia in 1947, where he fin-

ished secondary school, got a math degree from Yerevan State University, and taught high school for a year. He moved to Moscow in 1957, where he wrote a Kandidat thesis on lee winds downstream of a mountain with I. A. Kibel of Moscow State's department of dynamic meteorology, married a Russian, had a daughter, and worked from 1961 to 1965 at the Moscow Meteorological Computing Center. Taking advantage of a visit from DeGaulle, the family moved to Paris when Zeytounian was 38.

Zeytounian worked in the aerodynamics department at the French space agency ONERA from 1967 to 1972 and as a professor at the University of Lille-I from 1972 to 1996. He earned a doctorate in 1969, after his committee member Paul Germain forced him to justify the Boussinesq approximation via asymptotics. He learned matched asymptotic expansions from Jean-Pierre Guiraud and they planned books on rational asymptotic modeling (RAM), which ultimately failed to reach Guiraud's standard of perfection. Nonetheless, Zeytounian wrote many books on related topics and Guiraud kindly wrote the foreword to this one. The book outlines his unusual life and work, justifying some of it by quoting (even somewhat critical) reviews.

The most outstanding chapter develops the basic Navier–Stokes–Fourier equations in a historical context. Overall, readers will realize the unique value of Zeytounian's work and perspective and will come to appreciate his willingness to tackle very difficult problems, aiming to help fluid dynamical "numericians." An example is modeling turbomachinery using the number of blades on a rotor as a large parameter. Readers will need to overlook the author's difficulties in writing and Springer's lack of copyediting.

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Introduction to the Network Approximation Method for Materials Modeling. By L. Berlyand, A. G. Kolpakov, and A. Novikov. Cambridge University Press, Cambridge, UK, 2013. \$80.00. xiv+243 pp., hardcover. ISBN 978-1-107-02823-4.

The book introduces and analyzes a new effective method for the computation of the macroscopic properties of heterogeneous materials containing dispersed discs. This method is based on network approximation, i.e., the discs are replaced by their centers forming the so-called Delaunay–Voronoi graph, and the partial differential equation in the continuous, but heterogeneous, media is replaced by a natural discrete problem on the graph. This method is justified in the case of closely spaced perfectly conductive discs.

In physics and engineering, network models became classical after the book by Born and Huang [1] and have been intensively used ever since. The best-known example of the structural approximation is the “sprin” model used to describe a great variety of phenomena from molecular dynamics to composite materials; see examples in Sahimi [2].

Generally, the main tool of the multiscale modeling of composite materials is the homogenization method. However, this method is very sensitive to the periodicity of the inclusions: if the inclusions are not periodically spaced, then the result of the homogenization becomes much less “algorithmic.” This means that in nonperiodic cases, theoretically one can justify the existence of an equivalent homogeneous medium, but it is very difficult to compute its macroscopic properties.

Another issue complicating the application of the homogenization method is the presence of multiple small parameters in the problem, in particular, contrasting properties of the components of the composite material, thin regions spacing the discs, and a strong concentration of local fields in these regions. More specifically, the contrasting properties (such as heat or electrical conductivity) of the components may fall beyond the limits of applicability of the homogenization approach see [3].

That is why the rigorous analysis of the proposed methods is very important. In the present book the reader finds an excellent introduction to a new method that is applicable in exactly this most difficult case of a nonperiodic composite with contrasting properties, presented by the authors of this method.

The book’s preface discusses the relation between the discrete and continuous models. Chapter 1 reminds the reader of mathematical notions and facts used later in the book in the description and justification of the method of network approximations. This review is brief, but contains all necessary references for the reader who wants to study the question more deeply. Chapter 2 provides an introduction to the theory giving the motivation and some basic simple examples explaining the main idea of the method. Chapter 3 is the central one. It introduces the rigorous formulation of the problem and contains the complete proof of the main theorem (Theorem 3.14) justifying the network approximation formula (3.8.17) in the case of a finite number of closely spaced perfectly conductive inclusions. The main idea of the proof is the construction of upper and lower bounds for the effective coefficient in such a way that the leading terms of the asymptotic expansions coincide. The network approximations give an asymptotic solution to the problem of capacity of an arbitrary (random) system of many closely placed discs. The asymptotics of capacity may be computed from the solution to an algebraic $N \times N$ linear system, where N is the number of the disks.

The advantage of the book is that it contains not only theoretical results, but the numerical experiments as well, and Chapter 4 is devoted to numerics for percolation using the method of network approximations. It contains very interesting results for the effective conductivity. Chapter 5 considers the case of an infinite number of closely spaced perfectly conductive inclusions; it contains the rigorous formulation of the problem and the main theorem, which shows that the error of the network approximation is determined not by the total number of particles (as is usual in homogenization theory), but by the perimeter of the spaces between the groups of neighboring particles. Chapter 6 generalizes the network approximation approach to the case of nonlinear settings for the field equation (p -Laplacian). The previous chapters treated conductivity of the material “as a whole,” that is, as a characteristic integral. Chapter 7 studies local properties of the network approximation approach. It shows, in par-

ticular, that the potentials of the nodes in the network approximation corresponding to the centers of closely spaced perfectly conducting discs approximate the real potentials of the discs in the continuous model. Chapter 8 describes complex variable methods in the analysis of two-dimensional problems for closely spaced discs.

The book is well written. It gives an easy but detailed and comprehensive introduction to the subject. The methods are explained in simple examples. The proofs are supplied by physical and geometrical interpretations so that the book is comprehensible to a wide spectrum of readers who may not be specialists in the topic. In particular, it will be interesting for Ph.D. students in mathematics, physics, chemistry, or engineering. The price is low at around £50 in Cambridge University Press catalogue. Manufactured on demand, it is supplied directly from the printer and a second edition will be available in paperback.

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Number Theory. A Historical Approach.
 By John J. Watkins. Princeton University Press, Princeton, NJ, 2013. \$75.00. xvi+592 pp., hardcover. ISBN 978-0-691-15940-9.

This book could well have been titled *Number Theory and Its History*, if Oystein Ore hadn't already used that title for a book published in 1948. Ore's book is something of a classic, but I believe that the modern reader will find Watkins' book much more satisfying. It is far richer, deeper, and more comprehensive—which shows that number theory has become a much more main-

stream subject than it was in 1948. Not only has it become a standard undergraduate course, it is almost required reading for a public concerned about the security and privacy of the internet.

Watkins' *Number Theory* is ideal for a variety of readers: it is well suited to an undergraduate course and also as general reading for anyone from a bright high school student to a mathematician with an interest in number theory. In my opinion it maintains the perfect balance between mathematics and history: most of the theorems are proved in complete and rigorous fashion, as one would expect in a mathematics course, but the history explains where the ideas came from and why they were of interest to mathematicians. Also, of course, the history of number theory is particularly rich in characters and anecdotes (think Hardy, Ramanujan, and the number 1729).

The mathematical content includes the usual core topics: Pythagorean triples, existence and uniqueness of prime factorization, the Euclidean algorithm, linear and quadratic Diophantine equations (particularly the Pell equation), congruences, sums of squares, and quadratic reciprocity. These topics ensure that the book could be used as an undergraduate number theory textbook, but there are many other topics that enhance its flavor and usefulness: algorithms for factorization and primality testing, cryptography (more than just RSA), the Fibonacci numbers, continued fractions, and partitions.

The methods used are mostly elementary (though they grow in complexity as the book unfolds) with essentially no algebraic or analytic number theory. Thus, as Watkins says,

there is almost nothing in terms of prerequisites that readers need to bring along with them except enthusiasm and curiosity.

I wished at first that Watkins had taken the opportunity to describe the evolution of algebraic and analytic methods that took place in response to the evolution of number theory, but on reflection I think he is wise to stick to elementary methods. The book is already quite hefty, and it would have become overweight if algebra and analysis

were treated as full partners of number theory.

The book is written in a pleasant relaxed style, with frequent historical excursions that motivate the material and provide relief from the more technical passages. Topics are developed in roughly chronological order, starting with Fermat and visiting the highlights of his work and that of his successors Euler, Lagrange, and Gauss. There are also “flashbacks” to the work of Euclid, Diophantus, Fibonacci, and others who helped pave the way for Fermat and other modern number theorists. For example, Chapter 1 proves quite a meaty theorem of Fermat—that the area of an integer-sided right-angled triangle cannot be a square—with the help of a flashback to Euclid’s treatment of Pythagorean triples.

Watkins’ narrative style of generally moving forward, but with flashbacks, seems very effective. It allows the reader to reach advanced results quite quickly, but also to relax occasionally by going back to a more elementary level. Also, by allowing ideas to be revisited, it reinforces key ideas by returning to them again and again. For example, we see Euclid’s proof that there are infinitely many primes in Chapter 2, an extension of the idea to primes of the form $4n + 1$ in the exercises in Chapter 6, and a more sophisticated proof that there are infinitely many primes (by Euler) in Chapter 10. Likewise, the Fermat theorem in Chapter 1 is revisited in Chapter 7, where it is used to prove Fermat’s last theorem for fourth powers.

The historical content is very attractively presented and it will undoubtedly be a bonus for most readers. As far as I can see it is pretty accurate. The only slip I noticed was on page 335: naming Nicolas Bernoulli, rather than Daniel, as the discoverer of the so-called “Binet formula” for the Fibonacci numbers. However, I must register a mild protest against the lack of references. There are no references to primary historical sources (finding such a reference would have picked up the Bernoulli error above) and this is particularly annoying in the case of quotations. For example, we are told on page 253 that Gauss was amazed that Archimedes did not develop a decimal, or similar, notation for numbers, and that

Gauss said, “To what heights would science now be raised if Archimedes had made that discovery!” If true, this is a remarkable nugget of information about Gauss—but no source is given. (The quote also appears in E.T. Bell’s *Men of Mathematics*, and Bell gives no source either.)

The exercises are a particularly admirable feature. They are not just plentiful, and with plentiful hints and solutions, but well thought out and interesting. Very often, substantial proofs are organized into guided sequences of exercises. A notable example is Zagier’s notorious “One-sentence proof that every prime $p \equiv 1 \pmod{4}$ is a sum of two squares,” which appeared in the *American Mathematical Monthly* of 1990 and has caused many headaches since. On page 162 Watkins expands Zagier’s sentence into an eight-step exercise, spread over two and a half pages!

The book concludes with a brief introduction to the free Sage software—perhaps not sufficient to allow readers to start programming, but enough to whet the appetite—some well-annotated suggestions for further reading, and a useful pronunciation guide to the names of prominent number theorists. (We mathematicians tend to forget that the pronunciation of Leibniz, Euler, and Jacobi will not be obvious to our students.)

To sum up: this is a very rich, well-organized, and highly readable book. It should be accessible to a wide spectrum of readers and is particularly suitable for a first course in number theory.

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Numerical Methods with Worked Examples: MATLAB Edition. Second Edition.
By C. Woodford and C. Phillips. Springer, Dordrecht, 2012. \$59.95. x+256 pp., hardcover. ISBN 978-94-007-1365-9.

One approach in numerical methods textbooks, which may be considered the standard, is to present methods first with analysis followed by examples, either worked out by the author(s) or left as exercises to the reader. While this approach is certainly suitable for the reader whose inter-

ests are principally in scientific computing, the analysis may prove unnecessary or even discouraging to other readers. Thus, Woodford and Phillips focus more on the application and implementation of numerical methods instead of the analysis. Throughout the book, they present material in what they call the “problem-solution-discussion” order. For the reader who is looking to incorporate numerical methods into their field without diving too deep into the analysis, the authors’ order of presentation is very appealing.

The authors begin each chapter by outlining its purpose and/or aims and finish each one with a summary of what was learned, supported by numerous exercises for the reader. In this way, the book is nicely compartmentalized so that the reader may pick and choose chapters of interest without being confused by previously introduced notation. The first chapter provides a well-written introduction to MATLAB and includes variable creation, conditionals, flow control, and I/O. This ensures that all readers can take advantage of the many programming exercises provided throughout the book.

The following two chapters are on linear and nonlinear equations. The authors motivate each of these chapters effectively with examples. Gaussian elimination, pivoting, Gauss–Seidel iteration, the bisection, secant, and Newton’s methods are all nicely covered, while the analysis is kept to a level where the reader does not need linear algebra experience, aside from familiarity with matrices and vectors. The conjugate gradient method might have been worth mentioning after the authors introduced symmetric, positive-definite systems; however, maybe the corresponding analysis would have been too heavy. Additionally, it is worthy of note that there appears to be a typo in (2.45), where the authors seek to demonstrate the effect of round-off error by exchanging the rows in a linear system. The line should read $1.234x_1 + 0.996x_2 = 1.23$ if “System (1)” and “System (2)” are to be equivalent.

The book proceeds with a chapter on interpolation, motivated by the need for information between measured values in an experiment. The authors introduce polynomial interpolation using either Newton

divided differences to construct the polynomials or Neville’s method for tabulated data. They state that higher-order polynomials come with higher computation costs, no guarantee of accuracy, and mention that extrapolation can be a “risky proposition.” However, some discussion about the oscillatory nature of higher-order polynomials could have helped further illuminate the breakdown of high-order polynomial interpolation. They finish the chapter with some well worked through examples on splines, linear least squares, and polynomial least squares interpolation.

The next two chapters are on numerical integration and differentiation. For integration, the authors stay true to their focus on application over analysis by first presenting the trapezoid/trapezium and Simpson’s rules with a graphical interpretation. Only then do they introduce some analysis with error terms to explain why the errors in higher-order methods decrease faster. The concepts of Gaussian and adaptive quadratures are also introduced. For differentiation, the analysis is deeper. Taylor series are used to introduce finite differences for first and second derivatives. The finite differences themselves, derivation of error terms, and the use of Cauchy’s formula to avoid machine precision issues (through reformulating high-order derivatives into integrals) are all very well written, albeit a slight deviation from the authors’ focus on application. There is also a typo worth noting on page 121, where the remainder R_1 should be at most $h^2/2$ times $f''(\Psi)$ (not $h/2$ as printed). Otherwise, the error R_1/h would appear to be proportional to 1 instead of h .

The next two chapters are on linear programming and general optimization. The linear programming chapter is thoroughly motivated by word problems, including the ubiquitous Traveling Salesman, that are then translated into the appropriate form for the reader. The authors cover the concept of feasible regions and the details of the simplex method with graphical descriptions rather than rigorous proof, keeping the analysis to a minimum. Additionally, they cover integer programming and decision making, all accompanied by real-world examples, before moving to nonlinear optimization. This section is equally well motivated by physical

problems and their corresponding quantities to be optimized. Grid search methods are generalized to the higher-dimensional steepest descent and quasi-Newton methods. The authors close the chapter with penalty and Lagrangian approaches to constrained optimization, while keeping the linear algebra and vector calculus analysis to a minimum.

The authors include ODE numerical methods in the following chapter. After a nice introduction to how physical problems lead to equations to be solved, the authors cover both Euler and Runge–Kutta (second- and fourth-order) methods for initial value problems. The authors use computed error tables instead of lengthy truncation error analysis to show the order of the methods, while mentioning how the error would be determined. They include how to reformulate higher-order ODEs into first-order systems and how to then apply the methods. The chapter is wrapped up with a discussion on boundary value problems and how shooting-type or finite difference methods can be used. Although potentially outside the scope of this book, it might have been instructive to add some discussion on stability or step-size limitations.

The last two chapters are on numerical methods for finding the spectrum of a matrix and a light introduction to stochastic methods. In introducing the power and QR methods for finding eigenvectors and eigenvalues, the authors understandably dive a

little deeper into linear algebra than the previous chapters (including a simple proof of the linear independence of eigenspaces). This comes only after a nice motivation of why the spectrum is important in engineering, mainly dealing with resonance. The stochastic methods chapter introduces the concepts of distributions, expected value, variance, covariance, and correlation, along with how each one is computed. The authors also include introductory random number generation using the modulus operator and close the book with a description of Monte Carlo integration.

The chapters of this book cover almost everything that one would expect in a numerical methods book. The authors hold true to their endeavor to teach by example, including numerous motivating problems and worked solutions. They routinely choose graphical and intuitive motivation over proof and theory to effectively keep the reader focused on implementing the methods. The downside to this approach is that some aspects of the methods are lost in the balance of implementation with analysis. Thus, while this approach might leave some readers looking for more detailed explanation or mathematical motivation, it presents the book nicely to the reader more interested in seeing the methods work than understanding why they work.

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