MATH 2602

LINEAR AND DISCRETE MATHEMATICS

PROF. MARGALIT

WHAT IS DISCRETE MATH?

dis·crete [dih-**skreet**] Show IPA

adjective

1. apart or detached from others; separate; distinct: *six discrete parts.*
2. consisting of or characterized by distinct or individual parts; discontinuous.
3. *Mathematics* .
   - a. (of a topology or topological space) having the property that every subset is an open set.
   - b. defined only for an isolated set of points: *a discrete variable.*
   - c. using only arithmetic and algebra; not involving calculus: *discrete methods.*

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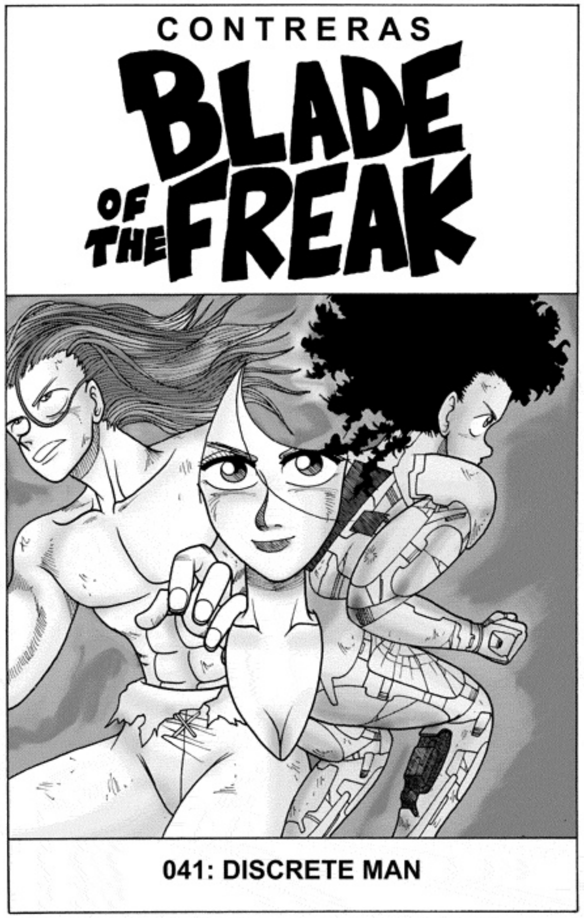
dictionary.com

Discrete is the opposite of continuous.

WHAT IS DISCRETE MATH?



WHAT IS DISCRETE MATH?

CONTINUOUS	DISCRETE
real numbers	integers
measuring	counting
ideal shapes	computer images
wave	particle
differential eqn	recurrence reln.
calculus	probability
	graph theory
	algorithms

CHAPTER 0

YES, THERE ARE PROOFS!

Knights and Knaves

Everyone is either a knight (truth-teller) or knave (liar).

1. Anna says Elsa is a knight.
   Else says she is a knight.
   What can you conclude?

2. Anna says at least one of us is a knave.
   What can you conclude?

0.1 Compound Statements

STATEMENTS

A mathematical statement is a declarative sentence that is either true or false.

Examples. 1 is a prime number.
π is a rational number.
If $1+1=3$ then $5=7$

Non-examples. What is my name?
Solve for x: $2x=10$.
Meep meep.
Et cetera.

We sometimes represent a statement by a letter.

THIS IS FALSE

Consider the following sentence:

This statement is false.

Is this a mathematical statement? Is it true or false?

NEW STATEMENTS FROM OLD

AND. $p \wedge q$ is true if both are.

OR. $p \vee q$ is true if at least one is.

CAUTION! Or has different uses in English: can have soup or salad
need a license or passport

NOT. $\neg p$ is true if p isn't.

"it is not the case that p"

IMPLICATIONS

IMPLICATION. $p \to q$ is true unless p true, q false.

"if p then q"

think about campaign promises

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

EXAMPLES. If $1=2$, I am the pope.
If I have 4 quarters, I have a dollar.

Implications

Converse. The converse of $p \to q$ is $q \to p$

The converse of a true statement can be true or false.

Contrapositive. The contrapositive of $p \to q$ is $\neg q \to \neg p$

The contrapositive is <u>equivalent</u> to the original statement!

If you won you got a medal.
If you didn't get a medal, you didn't win.

Negation. $\neg(p \to q) \equiv p \wedge \neg q$

Double Implication. $(p \to q) \wedge (q \to p)$ is written $p \leftrightarrow q$

You got a medal if and only if you won.

QUANTIFIERS

First, a propositional function is a statement with a variable that becomes a mathematical statement when a value is given to the variable:

n is even

We can also turn a propositional function into a mathematical statement using quantifiers.

For all. $\forall n \in \mathbb{Z}$ (n is even)
There exists. $\exists n \in \mathbb{Z}$ (n is even)

and combinations: $\forall n \in \mathbb{Z}\ \exists m \in \mathbb{Z}$ ($n+m$ is even)
$\exists m \in \mathbb{Z}\ \forall n \in \mathbb{Z}$ ($n+m$ is even) etc.

NEGATION. $\neg(\exists m \forall n\ (n+m \text{ is even})) \equiv$
$\forall m \exists n\ (n+m \text{ is odd})$

THE SECRET $\forall$

When we say:

If n is even, then $n+1$ is odd.

We really mean:

$$\forall n \, (n \text{ even} \rightarrow n+1 \text{ odd})$$

So for instance the negation is:

$$\exists n \; \neg (n \text{ even} \rightarrow n+1 \text{ odd})$$

$$\exists n \, (n \text{ even} \wedge n+1 \text{ even})$$