PRACTICE TEST 1 Name ____________________

Work neatly. Justify your answers and use proper notation. SHOW YOUR WORK TO RECEIVE CREDIT! No calculators or electronic devices are allowed.
There is a total of 100 points.

(14) 1. Consider the system of equations

$$\begin{aligned} x_1 + -3\,x_2 &= k \\ 2\,x_1 + h\,x_2 &= 10 \end{aligned}$$

Determine h and k (if possible) such that the solution set of the system

a. has more than one solution.

b. has exactly one solution.

Show all work and explain your answers.

(14) 2. Consider the two planes whose equations are

$$\begin{aligned} 4x_1 - 2x_2 + 7x_3 &= -5 \\ 4x_1 \ - x_2 + 3x_3 &= 2 \end{aligned}$$

a. These planes intersect. Find a vector parametric description of the solution set of the intersection.

b. Is the intersection of these two planes a point?, a line?, a plane?

(6) 3. Let $A = \begin{bmatrix} 5 & 2 & 3 & 4 \\ -2 & 0 & 3 & 3 \\ -2 & 1 & 2 & 2 \\ 6 & 1 & 3 & -1 \end{bmatrix}$ $\mathbf{v} = \begin{bmatrix} 2 \\ -5 \\ 3 \\ 7 \end{bmatrix}$. Find $A\mathbf{v}$.

(13) 4. Suppose A in a 3×5 matrix and can be row reduced to $\begin{bmatrix} 1 & 0 & 13 & 0 & -4 \\ 0 & 1 & 12 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$. Find the parametric vector form of the solutions set to $A\,\mathbf{x} = \mathbf{0}$.

(17) 5. In parts (a), (b), and (c) below, $\cdot$ denotes a nonzero entry and $*$ denotes an entry that may be 0 or nonzero.

5. (a). Suppose that $\begin{bmatrix} \cdot & * & * & * & * & * \\ 0 & \cdot & * & * & * & * \\ 0 & 0 & \cdot & * & * & * \\ 0 & 0 & 0 & 0 & \cdot & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is the row reduced augmented matrix corresponding to the equation $A\mathbf{x} = \mathbf{b}$.

(i) Does $A\mathbf{x} = \mathbf{b}$ have a solution? ________ Why? ________

(ii) If $A\mathbf{x} = \mathbf{b}$ has a solution, is this solution unique? ________

5. (b). Suppose that $\begin{bmatrix} \cdot & * & * & * & * & * \\ 0 & \cdot & * & * & * & * \\ 0 & 0 & \cdot & * & * & * \\ 0 & 0 & 0 & \cdot & * & * \\ 0 & 0 & 0 & 0 & \cdot & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is the row reduced augmented matrix corresponding to the equation $A\mathbf{x} = \mathbf{b}$.

(i) Does $A\mathbf{x} = \mathbf{b}$ have a solution? ________

(ii) If $A\mathbf{x} = \mathbf{b}$ has a solution, is this solution unique? ________

5. (c). Suppose that $\begin{bmatrix} \cdot & * & * & * & * & * & * \\ 0 & 0 & 0 & \cdot & * & * & * \\ 0 & 0 & 0 & 0 & \cdot & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdot \end{bmatrix}$ is the row reduced augmented matrix corresponding to the equation $A\mathbf{x} = \mathbf{b}$.

(i) Does $A\mathbf{x} = \mathbf{b}$ have a solution? ________ Why? ________

(ii) If $A\mathbf{x} = \mathbf{b}$ has a solution, is this solution unique? ________

(14) 6. Let $A = \begin{bmatrix} 1 & -4 & -1 \\ -1 & 4 & 1 \\ 0 & 2 & 1 \\ 2 & -2 & 0 \end{bmatrix}$,

a. Are the columns on A linearly independent? Justify your answer.
b. Do the columns of A span $I\!R^4$? Justify your answer.

(12) 7. Let $T : I\!R^3 \to I\!R^5$ be the linear transformation with

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_3 \\ 2x_1 + x_2 + 9x_3 \\ 4x_2 + 2x_3 \\ x_1 - 2x_2 \\ 5x_2 \end{bmatrix}$$

Find the matrix A so that $T(\mathbf{x}) = A\,\mathbf{x}$.

(11) 8. Mark the following statements as TRUE or FALSE or fill in the blank. You do not need to justify your answer.

a. __________ If the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}, \}$ of vectors in $I\!R^6$ is linearly dependent, then the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}, \mathbf{v_5}\}$ must also be linearly dependent.

b. __________ If the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ are vectors in $I\!R^5$, then the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ must be linearly independent.

c. __________ If A and B are $n \times n$ matrices, then $A\,B = B\,A$.

d. __________ A is a 5×7 matrix. The columns of A span R^5 if A has 5 pivot columns.

e. A is an $m \times n$ matrix. The columns of A are linearly independent if A has __________ pivot columns.