Name ________________________

Section HP ___

Mathematics 1553

Practice Midterm 3

Prof. Margalit

1. State the definition of an *eigenvector*.

State the definition of *diagonalizable*.

2. Is $(1, -1, -1)$ an eigenvector of

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{pmatrix}?$$

If so, find the eigenvalue.

Find a basis for the eigenspace with eigenvalue 2 for the matrix

$$A = \begin{pmatrix} 3 & -3 & 3 \\ -2 & 8 & -6 \\ -1 & 3 & -1 \end{pmatrix}.$$

3. Suppose that A is a 6×6 matrix and that it has an eigenvalue λ with algebraic multiplicity 3. What are the possible dimensions for the corresponding eigenspace?

Suppose that A is a 6×6 matrix and that it has an eigenvalue λ with algebraic multiplicity 6. Is A diagonalizable? Answer *yes / no / maybe*.

Suppose that A is a 6×6 matrix and that it has an eigenvalues 1, 2, 3, 4, 5, and 6. Is A diagonalizable? Answer *yes / no / maybe*.

4. Find the characteristic polynomial for the following matrix:

$$A = \begin{pmatrix} 3 & 0 & 0 \\ -1 & 1 & 1 \\ 2 & 2 & -5 \end{pmatrix}$$

What are the eigenvalues of A?

Compute the determinant of the following matrix:

$$A = \begin{pmatrix} 3 & 0 & 0 & 4 \\ 5 & 8 & 3 & -8 \\ 2 & 0 & 0 & 0 \\ 4 & 2 & 1 & 1 \end{pmatrix}$$

What is the determinant of A^2?

5. Compute the determinant of the following matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Find the determinant of the following matrix by row reduction. Show all of your steps.

$$A = \begin{pmatrix} 1 & 5 & -6 \\ -1 & -4 & -5 \\ -2 & -8 & 7 \end{pmatrix}$$

6. Find the area of the triangle Δ with vertices $(2,1)$, $(-1,5)$, and $(3,10)$.

Now consider the same Δ and the matrix.

$$A = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$

What is the area of $T_A(\Delta)$?

7. Use determinants to determine whether or not these three vectors linearly independent.

$$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

Find two solutions to the following difference equation:

$$x_{k+1} = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} x_k$$

8. Consider the matrix

$$A = \begin{pmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & -3 \end{pmatrix}$$

Is A diagonalizable? If so, diagonalize it. If not, explain why it is not diagonalizable.

Ask your friend (or enemy) to make up a diagonalization problem for you. Here's how. They choose a matrix B that is either diagonal or not. Then they choose an invertible matrix C and tell you the product $A = CBC^{-1}$ (without telling you what B and C are). Then you figure out if A is diagonalizable or not.

9. Use Cramer's rule to solve:

$$\begin{aligned} x_1 + x_2 &= 4 \\ -5x_1 + 2x_3 &= 0 \\ x_2 - 2x_3 &= 3 \end{aligned}$$

Compute the adjugate of the matrix

$$A = \begin{pmatrix} 0 & -3 & -1 \\ 2 & 0 & 0 \\ -2 & 1 & 2 \end{pmatrix}$$

Remember that the adjugate of a matrix is the matrix whose ijth entry is the jith cofactor of A.

10. A rental car agency has two locations. One fourth of the cars from the first location get returned to the first location and three-fourths to the second. Two-thirds of the cars from the second location get returned to the first location and one-third to the second. How should the agency distribute their cars in order to minimize the number of cars that need to be shuttled?

Explain why similar matrices have the same eigenvalues.