Name KEY

Mathematics 1553

Quiz 10
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Section HP1 / HP2
20 November 2015

1. Find the length of the vector $(1, 2, 2)$ in $\mathbb{R}^3$.

$$\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{1+2^2+2^2} = \sqrt{9} = \boxed{3}$$

Suppose that L is the span of $(1, 2, 2)$ in $\mathbb{R}^3$. What is the projection of $(1, 0, 0)$ to L? (Your answer should be a vector.)

$$\vec{V}_L = \frac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\cdot\vec{u} = \frac{1^2+0\cdot 2+0\cdot 2}{1^2+2^2+2^2}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}$$

$$= \frac{1}{9}\begin{pmatrix}1\\2\\2\end{pmatrix} = \boxed{\begin{pmatrix}\frac{1}{9}\\ \frac{2}{9}\\ \frac{2}{9}\end{pmatrix}}$$

Give a basis for the orthogonal complement to L (the span of $(1, 2, 2)$).

All vectors in the orthogonal complement to L, satisfy the property of $\vec{v}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix} = 0$

So Find the null space of matrix $\begin{pmatrix}1 & 2 & 2\end{pmatrix}$

$$\left(\begin{array}{ccc|c}1 & 2 & 2 & 0\end{array}\right) \Rightarrow \begin{cases} x_1 = -2x_2 - 2x_3 \\ x_2 = x_2 \\ x_3 = x_3 \end{cases}$$

$$\Rightarrow \vec{v} = x_2\begin{pmatrix}-2\\1\\0\end{pmatrix} + x_3\begin{pmatrix}-2\\0\\1\end{pmatrix}$$ So basis is $\left\{\begin{pmatrix}-2\\1\\0\end{pmatrix}, \begin{pmatrix}-2\\0\\1\end{pmatrix}\right\}$