

Mathematics 1553

Quiz 2

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Section HP1 / HP2

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1. Solve the matrix equation

$$\begin{pmatrix} 1 & 0 & -5 \\ 0 & 4 & -3 \\ -2 & 8 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 10 \\ 2 \end{pmatrix}$$

$$\begin{aligned} \left(\begin{array}{ccc|c} 1 & 0 & -5 & 11 \\ 0 & 4 & -3 & 10 \\ -2 & 8 & 2 & 2 \end{array} \right) &\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -5 & 11 \\ 0 & 4 & -3 & 10 \\ 0 & 8 & -8 & 24 \end{array} \right) &\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -5 & 11 \\ 0 & 4 & -3 & 10 \\ 0 & 1 & -1 & 3 \end{array} \right) &\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -5 & 11 \\ 0 & 0 & 1 & -2 \\ 0 & 1 & -1 & 3 \end{array} \right) \\ &\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -5 & 11 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right) &\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right) && \boxed{\begin{array}{l} x=1 \\ y=1 \\ z=-2 \end{array}} \end{aligned}$$

Your solution to the first part allows you to write $(11, 10, 2)$ as a linear combination of three other vectors. Write this linear combination.

$$1 \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 4 \\ 8 \end{pmatrix} - 2 \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 10 \\ 2 \end{pmatrix}$$

For which vectors (b_1, b_2, b_3) does

$$\begin{pmatrix} 1 & 0 & -5 \\ 0 & 4 & -3 \\ -2 & 8 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

have a solution? Explain your answer.

$$A = \begin{pmatrix} 1 & 0 & -5 \\ 0 & 4 & -3 \\ -2 & 8 & 2 \end{pmatrix}$$

Because the RREF of A has a pivot in every column, the matrix equation will have a solution for every vector (b_1, b_2, b_3) in \mathbb{R}^3 .