Name Key

Mathematics 1553

Quiz 4

Prof. Margalit

Section HP1 / HP2

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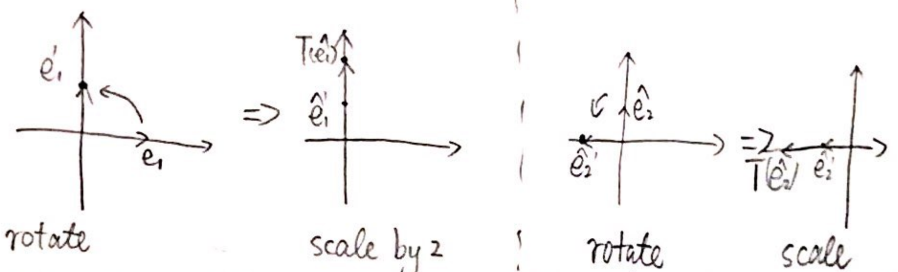
1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotates counterclockwise by $\pi/2$ and then scales by a factor of two in all directions. Write down the standard matrix for T.

$T(\hat{e}_1) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$T(\hat{e}_2) = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$



$T = \begin{pmatrix} T(\hat{e}_1) & T(\hat{e}_2) \end{pmatrix}$

$= \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

Show the possible row echelon form(s) for the standard matrix of a one-to-one linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$. Use squares for pivot positions, stars for arbitrary numbers that are not pivots, and zeros for zeros.

$T: \mathbb{R}^2 \to \mathbb{R}^2 \Rightarrow 2 \times 2$ matrix

one to one $\Rightarrow$ every column has a pivot.

$\hookrightarrow \begin{pmatrix} \square & * \\ 0 & \square \end{pmatrix}$

True/False. A one-to-one linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is necessarily onto as well. Explain your answer.

True. Assume the transformation is represented by matrix A. Since it's one-to-one, ever column of A has a pivot, in total of 2 pivots, which means every row of A has a pivot, Therefore the columns of A span $\mathbb{R}^2$, and T is also onto.