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Mathematics 1553

Quiz 9
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Section HP1 / HP2
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1. Consider the matrix

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}.$$

Is $(1, 1, 1)$ an eigenvector for A? Why or why not?

$$\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

So $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of A with eigenvalue of 0.

List all eigenvalues of the following matrix:

triangular matrix

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & -1 \end{pmatrix}.$$

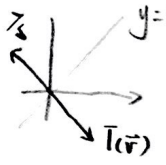
$$\det(A - \lambda I) = \det\begin{pmatrix} 1-\lambda & -1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & -1-\lambda \end{pmatrix} = (1-\lambda)(2-\lambda)(-1-\lambda) = 0$$

$$\Rightarrow \boxed{\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = -1}$$

Suppose A is a 2×2 matrix and that T_A is the linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$ that reflects about the line $y = x$. List one eigenvector of A and give the corresponding eigenvalue.

① if $\vec{v}$ is on the line $y=x$, after it is reflected, $\vec{v}$ stays the same.
So $\lambda_1 = 1$, $\vec{v_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, or any multiple of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

② if $\vec{v}$ is perpendicular to line $y=x$ after reflected, $T(\vec{v}) = -\vec{v}$



So $\lambda_2 = -1$, $\vec{v_2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, or any multiple.