Name SOLUTIONS

Mathematics 1553

Midterm 1

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1. Answer the following questions. No justification for your answer is required.

Is the matrix $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ in reduced row echelon form?

YES NO

Give an example of a matrix A so the solutions to $Ax = 0$ form a line in $\mathbb{R}^4$.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Suppose I have two linear equations in three variables. Which of the following are possible solutions to the system? Select all that apply.

(a) the empty set

(b) a single point

(c) a line

(d) a plane

(e) $\mathbb{R}^3$

Which of the following statements are equivalent to the statement that the matrix equation $Ax = b$ is consistent? Select all that apply.

(a) the augmented matrix $(A|b)$ has no pivot in the last column

(b) the augmented matrix $(A|b)$ has a pivot in every row

(c) b is a linear combination of the columns of A

(d) b lies in the span of the columns of A

2. Answer the following questions. No justification for your answer is required.

Give the definition of $\text{Span}\{v_1, \ldots, v_k\}$.

The set of linear combinations of $V_1, \ldots, V_k$

Say that A is an $m \times n$ matrix. Which of the following statements are equivalent to the statement that the columns of A span $\mathbb{R}^m$? Select all that apply.

(a) A has a pivot in each row

(b) A has a pivot in each column

(c) $Ax = b$ is consistent for every b in $\mathbb{R}^m$

(d) $Ax = 0$ is consistent

If the augmented matrix $(A|b)$ has more columns than pivots then the matrix equation $Ax = b$ has infinitely many solutions.

TRUE FALSE

Suppose A is a 2×2 matrix and the set of solutions to $Ax = 0$ is the line $y = x$. Also suppose that b is a nonzero vector in $\mathbb{R}^2$. Which of the following can possibly be the set of solutions to $Ax = b$? Select all that apply.

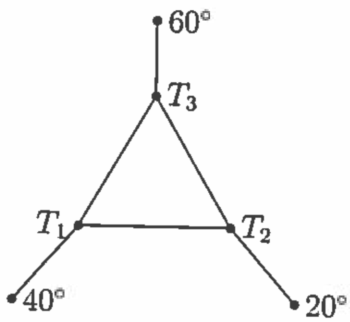
(a) the line $y = x$

(b) the line $y = x + 1$

(c) the line $x = 0$

(d) the origin

3. In a grid of wires, the temperature at exterior mesh points is maintained at constant values as shown in the figure. When the grid is in thermal equilibrium, the temperature at each interior mesh point is the average of the temperatures at the three adjacent points.



Suppose we would like to find the temperatures T_1, T_2, and T_3 when the grid is in equilibrium. Write a system of linear equations that determines the values of T_1, T_2, and T_3. Do not solve.

$$3T_1 = T_2 + T_3 + 40$$
$$3T_2 = T_3 + T_1 + 20$$
$$3T_3 = T_1 + T_2 + 60$$

Write the above system of linear equations as a vector equation. Do not solve.

$$T_1 \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} + T_2 \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix} + T_3 \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 40 \\ 20 \\ 60 \end{pmatrix}$$

Write the above system of linear equations as a matrix equation. Do not solve.

$$\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} 40 \\ 20 \\ 60 \end{pmatrix}$$

4. Consider the matrix equation $Ax = b$ where

$$A = \begin{pmatrix} 0 & 1 & 2 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & 3 & 8 & 0 \end{pmatrix} \text{ and } b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Find the reduced row echelon form of the augmented matrix $(A\,|\,b)$.

$$\left(\begin{array}{cccc|c} 0 & 1 & 2 & 4 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 1 & 3 & 8 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & 3 & 8 & 0 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 4 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & 0 & 2 & -3 & -2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cccc|c} 1 & 0 & 2 & -3 & -2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & 0 & 2 & 0 & -2 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

Write the set of solutions to $Ax = b$ in vector parametric form.

$$\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ -2 \\ 1 \\ 0 \end{pmatrix}$$

Write the set of solutions to $Ax = 0$ in vector parametric form.

$$x_3 \begin{pmatrix} -2 \\ -2 \\ 1 \\ 0 \end{pmatrix}$$

5. For which value of h does the vector

$$\begin{pmatrix} 2 \\ 7 \\ h \end{pmatrix}$$

lie in the span of the vectors

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}?$$

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 1 & 7 \\ 1 & -1 & h \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -1 & 3 \\ 0 & -2 & h-2 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & -3 \\ 0 & -2 & h-2 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & h-8 \end{array}\right) \qquad h = 8$$

Give the specific linear combination of $(1, 2, 1)$ and $(1, 1, -1)$ that equals your vector $(2, 7, h)$.

$$5 \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + (-3) \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ 8 \end{pmatrix}$$