1. Answer each of the following questions. You do not need to explain your answer.

Suppose that $A$ is an $n \times n$ matrix that is not invertible. Let $T$ be the linear transformation $T(v) = Av$. Which of the following can you conclude? Select all that apply.

(a) $A$ has two identical columns
(b) There is a vector $b$ so that $Ax = b$ has infinitely many solutions
(c) $A$ has a row of zeros
(d) There are two different vectors $u$ and $v$ in $\mathbb{R}^n$ with $T(u) = T(v)$
(e) The reduced row echelon form is not the identity

Consider the function $T : \mathbb{R} \to \mathbb{R}$ given by the formula $T(x) = x + 1$. Is $T$ a linear transformation?

YES

NO

Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by reflection about the line $y = 2x$. Is $T$ invertible?

YES

NO

Let $V$ be the first quadrant of $\mathbb{R}^2$. In other words $V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x \geq 0 \text{ and } y \geq 0 \right\}$. Is $V$ a subspace of $\mathbb{R}^2$?

YES

NO
2. Answer each of the following questions. You do not need to explain your answer.

Suppose that \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) be the linear transformation with

\[
T \left( \begin{array}{c} -1 \\ 1 \end{array} \right) = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \quad \text{and} \quad T \left( \begin{array}{c} 10 \\ 0 \end{array} \right) = \left( \begin{array}{c} 5 \\ 3 \end{array} \right)
\]

What is \( T \left( \begin{array}{c} 9 \\ 1 \end{array} \right) \)?

Suppose that \( A \) is a \( 2 \times 3 \) matrix and that the linear transformation \( T(v) = Av \) is onto. Describe the solutions of \( Ax = 0 \).

(a) a line in \( \mathbb{R}^2 \)
(b) \( \mathbb{R}^2 \)
(c) a line in \( \mathbb{R}^3 \)
(d) a plane in \( \mathbb{R}^3 \)
(e) none of the above

Assume that \( A, B, C, \) and \( X \) are invertible \( n \times n \) matrices. Solve for \( X \).

\[
XA + B = C
\]

What is the inverse of the matrix \( \left( \begin{array}{cc} 2 & 1 \\ 1 & 1 \end{array} \right) \)?
3. Complete this definition: a set of vectors \( \{v_1, \ldots, v_k\} \) is *linearly independent* if...

Find all values of \( k \) so that the following set of vectors is linearly dependent.

\[
\left\{ \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ k \\ -7 \end{pmatrix} \right\}
\]
4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ 0 \end{pmatrix}.$$ 

and let $U : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection over the $x$-axis.

What is the matrix for $T$?

What is the matrix for $U$?

Is $T$ one-to-one? YES NO

What is the matrix for $T \circ U$?

What is the range of $T \circ U$?
5. Consider the following matrix and its reduced row echelon form:

\[
A = \begin{pmatrix}
1 & -2 & 4 \\
0 & 0 & 1 \\
1 & -2 & 3 \\
-2 & 4 & -8 \\
\end{pmatrix} \sim \begin{pmatrix}
1 & -2 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix}
\]

Find a basis for \( \text{Nul}(A) \).

What is the dimension of \( \text{Col}(A) \)?

Find a basis \( B \) for \( \text{Col}(A) \).

What is

\[
\begin{bmatrix}
5 \\
1 \\
4 \\
-10 \\
\end{bmatrix}_B
\]
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<thead>
<tr>
<th>Problem</th>
<th>Score</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
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