Name SOLUTIONS

Mathematics 1553

Midterm 2

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1. Answer each of the following questions. You do not need to explain your answer.

Suppose that A is an $n \times n$ matrix that ***is not*** invertible. Let T be the linear transformation $T(v) = Av$. Which of the following can you conclude? Select all that apply.

(a) A has two identical columns

(b) There is a vector b so that $Ax = b$ has infinitely many solutions

(c) A has a row of zeros

(d) There are two different vectors u and v in $\mathbb{R}^n$ with $T(u) = T(v)$

(e) The reduced row echelon form is not the identity

Consider the function $T : \mathbb{R} \to \mathbb{R}$ given by the formula $T(x) = x + 1$. Is T a linear transformation?

YES NO

Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by reflection about the line $y = 2x$. Is T invertible?

YES NO

Let V be the first quadrant of $\mathbb{R}^2$. In other words $V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x \geq 0 \text{ and } y \geq 0 \right\}$.

Is V a subspace of $\mathbb{R}^2$?

YES NO

2. Answer each of the following questions. You do not need to explain your answer.

Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with

$$T\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } T\begin{pmatrix} 10 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

What is $T\begin{pmatrix} 9 \\ 1 \end{pmatrix}$?

$$\begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

Suppose that A is a 2×3 matrix and that the linear transformation $T(v) = Av$ is onto. Describe the solutions of $Ax = 0$.

(a) a line in $\mathbb{R}^2$
(b) $\mathbb{R}^2$
(c) a line in $\mathbb{R}^3$
(d) a plane in $\mathbb{R}^3$
(e) none of the above

Assume that A, B, C, and X are invertible $n \times n$ matrices. Solve for X.

$$XA + B = C$$

$$X = (C-B)A^{-1}$$

What is the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$?

$$\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

3. Complete this definition: a set of vectors $\{v_1, \ldots, v_k\}$ is *linearly independent* if...

the only Solution to

$$c v_1 + \cdots + c_k v_k = 0$$

is the trivial one.

Find all values of k so that the following set of vectors is linearly dependent.

$$\left\{ \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ k \\ -7 \end{pmatrix} \right\}$$

$$\begin{pmatrix} -1 & 1 & 1 \\ 3 & 1 & k \\ -1 & -1 & -7 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -1 & -1 \\ 3 & 1 & k \\ -1 & -1 & 7 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 1 & -1 & -1 \\ 0 & 4 & k+3 \\ 0 & -2 & -8 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 4 \\ 0 & 4 & k+3 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & k-13 \end{pmatrix}$$

$$\boxed{k = 13}$$

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ 0 \end{pmatrix}.$$

and let $U : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection over the x-axis.

What is the matrix for T?

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

What is the matrix for U?

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Is T one-to-one? YES NO

What is the matrix for $T \circ U$?

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

What is the range of $T \circ U$?

X-axis

5. Consider the following matrix and its reduced row echelon form:

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 0 & 1 \\ 1 & -2 & 3 \\ -2 & 4 & -8 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Find a basis for Nul(A).

$$\begin{aligned} x_1 &= 2x_2 \\ x_2 &= x_2 \\ x_3 &= 0 \end{aligned} \rightsquigarrow x_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \rightsquigarrow \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$$

What is the dimension of Col(A)?

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Find a basis B for Col(A).

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 3 \\ -8 \end{pmatrix} \right\}$$

What is

$$\left[\begin{pmatrix} 5 \\ 1 \\ 4 \\ -10 \end{pmatrix} \right]_B ?$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(the given vector is the sum of the basis elements)