Mathematics 1553

Midterm 3

Prof. Margalit

Section   E1/Arjun   E2/Qianli   E3/Kemi   E4/Martin   E5/Bharat   (circle one!)

17 November 2017
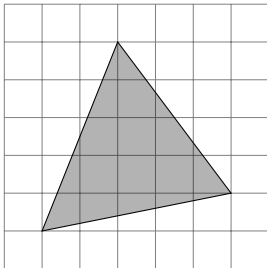
1. Answer each of the following questions. You do not need to explain your answer.

Compute the determinant.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

Find the area of the shaded triangle.

Suppose that $A = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$, that $T(v) = Av$, and that S is a region of $\mathbb{R}^2$ with area π.
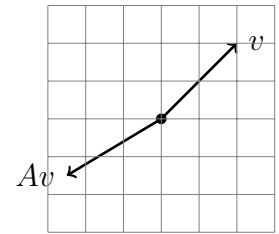
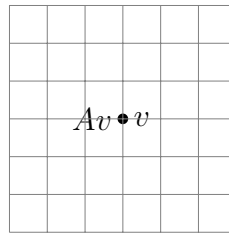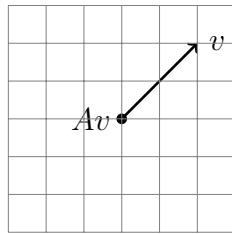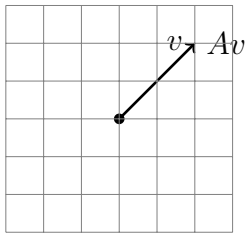What is the area of $T(S)$?

If A is an $n \times n$ matrix and $\det(A) = 5$ then $\det(-A) = 5$.

TRUE   FALSE

2. Answer each of the following questions. You do not need to explain your answer.

Under each picture, write the *eigenvalue* being depicted. If the picture does not depict an eigenvector, write NO. (Only real eigenvalues allowed.)

Suppose that T is a linear transformation of $\mathbb{R}^2$ given by reflection about the line $y = 5x$ and that $T(v) = Av$. What are the eigenvalues of A?

Every 4×4 matrix has at least one real eigenvalue.

        TRUE   FALSE

If 3 is an eigenvalue of A then 0 is an eigenvalue of $A - 3I$.

        TRUE   FALSE

3. Consider the matrix $A = \begin{pmatrix} 6 & -10 \\ 4 & -6 \end{pmatrix}$.

Find the eigenvalues of A.

Find an eigenvector for the eigenvalue with negative imaginary part.

Find a rotation + scaling matrix B to which A is similar.

By how much does B scale?

By how much does B rotate?

Find a matrix C so that $A = CBC^{-1}$

4. Consider the matrix $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$.

Find the eigenvalues of A.

Find a basis for one eigenspace for A.

Find a basis for the other eigenspace for A.

Is A diagonalizable? If so give a diagonalization; otherwise, explain why it is not.

5. Answer each of the following questions. You do not need to explain your answers.

Give an example of a 2×2 matrix that is neither invertible nor diagonalizable.

Given an example of a 2×2 matrix with eigenvalues 0 and 1 and where the 0-eigenspace is the line $y = -x$ and the 1-eigenspace is the x-axis.

Given that
$$\begin{pmatrix} 4 & -10 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}^{-1},$$
find $\begin{pmatrix} 4 & -10 \\ 2 & -5 \end{pmatrix}^{99}$. Your answer should be a 2×2 matrix.

Problem	Score
1	
2	
3	
4	
5	
Total	