Name SOLUTIONS

Mathematics 1553

Midterm 3

Prof. Margalit

Section E1/Arjun E2/Qianli E3/Kemi E4/Martin E5/Bharat (circle one!)

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1. Answer each of the following questions. You do not need to explain your answer.

Compute the determinant.

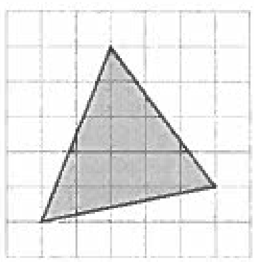
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

1 row swap

1 row rep

$\boxed{-6}$

Find the area of the shaded triangle.



$\frac{1}{2} \det \begin{pmatrix} 5 & 2 \\ 1 & 5 \end{pmatrix} = 23/2$

Suppose that $A = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$, that $T(v) = Av$, and that S is a region of $\mathbb{R}^2$ with area π. What is the area of $T(S)$?

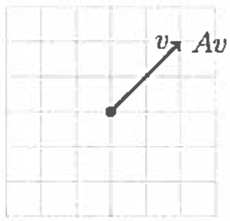
$|\det A| \pi = \pi$

If A is an $n \times n$ matrix and $\det(A) = 5$ then $\det(-A) = 5$.

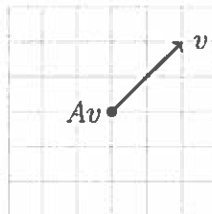
TRUE FALSE

2. Answer each of the following questions. You do not need to explain your answer.

Under each picture, write the *eigenvalue* being depicted. If the picture does not depict an eigenvector, write NO. (Only real eigenvalues allowed.)

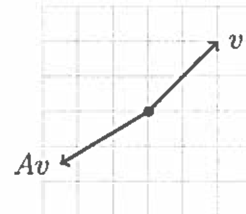


1



0

NO



NO

Suppose that T is a linear transformation of $\mathbb{R}^2$ given by reflection about the line $y = 5x$ and that $T(v) = Av$. What are the eigenvalues of A?

± 1

Every 4×4 matrix has at least one real eigenvalue.

TRUE FALSE

If 3 is an eigenvalue of A then 0 is an eigenvalue of $A - 3I$.

TRUE FALSE

3. Consider the matrix $A = \begin{pmatrix} 6 & -10 \\ 4 & -6 \end{pmatrix}$.

Find the eigenvalues of A.

$$\det \begin{pmatrix} 6-\lambda & -10 \\ 4 & -6-\lambda \end{pmatrix} = -(6-\lambda)(6+\lambda) + 40$$

$$= \lambda^2 + 4$$

$$\boxed{\pm 2i}$$

Find an eigenvector for the eigenvalue with negative imaginary part.

$$\begin{pmatrix} 6+2i & -10 \\ 0 & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 10 \\ 6+2i \end{pmatrix}$$

Find a rotation + scaling matrix B to which A is similar. $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

By how much does B scale? 2

By how much does B rotate? $\pi/2$

Find a matrix C so that $A = CBC^{-1}$

$$\begin{pmatrix} 10 & 0 \\ 6 & 2 \end{pmatrix}$$

4. Consider the matrix $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$.

Find the eigenvalues of A.

$$\det \begin{pmatrix} 2-\lambda & 3 & 1 \\ 3 & 2-\lambda & 4 \\ 0 & 0 & -1-\lambda \end{pmatrix} = -(\lambda+1)\left((2-\lambda)^2 - 9\right)$$

$$= -(\lambda+1)(\lambda^2 - 4\lambda - 5)$$

$$= -(\lambda+1)(\lambda-5)(\lambda+1)$$

$$\lambda = -1, 5$$

Find a basis for one eigenspace for A.

$\lambda = 5$

$$\begin{pmatrix} -3 & 3 & 1 \\ 3 & -3 & 4 \\ 0 & 0 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

Find a basis for the other eigenspace for A.

$\lambda = -1$

$$\begin{pmatrix} 3 & 3 & 1 \\ 3 & 3 & 4 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\}$$

Is A diagonalizable? If so give a diagonalization; otherwise, explain why it is not.

No. geom mult. of -1 $<$ alg. mult. of -1

5. Answer each of the following questions. You do not need to explain your answers.

Give an example of a 2×2 matrix that is neither invertible nor diagonalizable.

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Given an example of a 2×2 matrix with eigenvalues 0 and 1 and where the 0-eigenspace is the line $y = -x$ and the 1-eigenspace is the x-axis.

$$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

Given that

$$\begin{pmatrix} 4 & -10 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}^{-1},$$

find $\begin{pmatrix} 4 & -10 \\ 2 & -5 \end{pmatrix}^{99}$. Your answer should be a 2×2 matrix.

$$\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & -1^{99} \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}^{-1}$$

$$\begin{pmatrix} 4 & -10 \\ 2 & -5 \end{pmatrix}$$