MATH 1553
SAMPLE MIDTERM 2: 1.7-1.9, 2.1-2.3, 2.8-2.9

Name		Section	

1	2	3	4	5	Total

Please **read all instructions** carefully before beginning.

- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is intended to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

The exam is not designed to test material from the previous midterm on its own. However, knowledge of the material prior to section §1.7 is necessary for everything we do for the rest of the semester, so it is fair game for the exam as it applies to §§1.7 through 2.9.

Problem 1.

[2 points each]

Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.

a) **T** **F** If A is an $n \times n$ matrix and its rows are linearly independent, then $Ax = b$ has a unique solution for every b in $\mathbf{R}^n$.

b) **T** **F** If A is an $n \times n$ matrix and $Ae_1 = Ae_2$, then A is not invertible.

c) **T** **F** The solution set of a consistent matrix equation $Ax = b$ is a subspace.

d) **T** **F** If A and B are matrices and AB is invertible, then A and B are invertible.

e) **T** **F** There exists a 3×5 matrix with rank 4.

Problem 2.

Parts (a) and (b) are unrelated.

a) Find all values of x so that the matrix $\begin{pmatrix} -1 & 2-x \\ x & 3 \end{pmatrix}$ is invertible.

b) Consider $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ x+z \\ 3x-4y+z \end{pmatrix}.$$

Is T invertible? Justify your answer.

Problem 3.

a) Determine which of the following transformations are linear.

(1) $S : \mathbf{R}^2 \to \mathbf{R}^2$ given by $S(x_1, x_2) = (x_1,\ 3 + x_2)$

(2) $T : \mathbf{R}^2 \to \mathbf{R}^2$ given by $T(x_1, x_2) = (x_1 - x_2,\ x_1 x_2)$

(3) $U : \mathbf{R}^2 \to \mathbf{R}^3$ given by $U(x_1, x_2) = (-x_2,\ x_1, 0)$

b) Complete the following definition (be mathematically precise!):
A set of vectors $\{v_1, v_2, \dots, v_p\}$ in $\mathbf{R}^n$ is *linearly independent* if...

c) If $\{v_1, v_2, v_3\}$ are vectors in $\mathbf{R}^3$ with the property that none of the vectors is a scalar multiple of another, is $\{v_1, v_2, v_3\}$ necessarily linearly independent? Justify your answer.

Problem 4.

Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation which projects onto the yz-plane and then forgets the x-coordinate, and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation of rotation counterclockwise by 60°. Their standard matrices are

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{and} \qquad B = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix},$$

respectively.

a) Which inverse makes sense / exists? (Circle one.)

$$T^{-1} \qquad U^{-1}$$

b) Find the standard matrix for the inverse you circled in (a).

c) Which composition makes sense? (Circle one.)

$$U \circ T \qquad T \circ U$$

d) Find the standard matrix for the transformation that you circled in (c).

Problem 5.

Consider the following matrix A and its reduced row echelon form:

$$\begin{pmatrix} 2 & 4 & 7 & -16 \\ 3 & 6 & -1 & -1 \\ 5 & 10 & 6 & -17 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

a) Find a basis for $\operatorname{Col} A$.

b) Find a basis $\mathcal{B}$ for $\operatorname{Nul} A$.

c) For each of the following vectors v, decide if v is in $\operatorname{Nul} A$, and if so, find $[x]_{\mathcal{B}}$:

$$\begin{pmatrix} 7 \\ 3 \\ 1 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} -5 \\ 2 \\ -2 \\ -1 \end{pmatrix}$$

[Scratch work]