

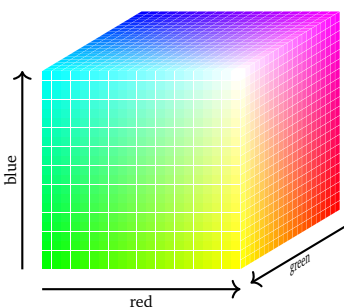
Math 1553 Worksheet §1.7, 1.8, 1.9

1. Justify why each of the following true statements can be checked without row reduction.

a) $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix} \right\}$ is linearly independent.

b) $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is linearly dependent.

2. Every color on my computer monitor is a vector in \mathbf{R}^3 with coordinates between 0 and 255, inclusive. The coordinates correspond to the amount of red, green, and blue in the color.






Given colors v_1, v_2, \dots, v_p , we can form a “weighted average” of these colors by making a linear combination

$$v = c_1 v_1 + c_2 v_2 + \dots + c_p v_p$$

with $c_1 + c_2 + \dots + c_p = 1$. Example:

$$\frac{1}{2} \text{ (red square)} + \frac{1}{2} \text{ (blue square)} = \text{ (purple square)}$$

Consider the colors on the right. Are these colors linearly independent? What does this tell you about the colors?

$\begin{pmatrix} 240 \\ 140 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 120 \\ 100 \end{pmatrix}$	$\begin{pmatrix} 60 \\ 125 \\ 75 \end{pmatrix}$
		

3. Let A be a 3×4 matrix with column vectors v_1, v_2, v_3, v_4 . Suppose that $v_2 = 2v_1 - 3v_4$. Find one non-trivial solution to the equation $Ax = 0$.

4. Which of the following transformations T are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the matrix is not one-to-one, find two vectors with the same image.

a) Counterclockwise rotation by 32° in \mathbf{R}^2 .

b) The transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x, y, z) = (z, x)$.

c) The transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x, y, z) = (0, x)$.

d) The matrix transformation with standard matrix $A = \begin{pmatrix} 1 & 6 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$.

e) The matrix transformation with standard matrix $A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

5. For each matrix A , describe what the associated matrix transformation T does to \mathbf{R}^3 geometrically.

$$\text{a) } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{b) } \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

6. The second little pig has decided to build his house out of sticks. The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of 45° in a counterclockwise direction about the z -axis, and then projected onto the xy -plane. Find the matrix for this transformation.