1. If $A$ is a $3 \times 5$ matrix and $B$ is a $3 \times 2$ matrix, which of the following are defined?
   a) $A - B$
   b) $AB$
   c) $A^T B$
   d) $B^T A$
   e) $A^2$

2. Find all matrices $B$ that satisfy
   \[
   \begin{pmatrix}
   1 & -3 \\
   -3 & 5
   \end{pmatrix} B = \begin{pmatrix}
   -3 & -11 \\
   1 & 17
   \end{pmatrix}.
   \]

3. a) If the columns of an $n \times n$ matrix $Z$ are linearly independent, is $Z$ necessarily invertible? Justify your answer.
   b) Solve $AB = BC$ for $A$, assuming $A, B, C$ are $n \times n$ matrices and $B$ is invertible. Be careful!

4. True or false (justify your answer). Answer true if the statement is always true. Otherwise, answer false.
   a) If $A$ is an $m \times n$ matrix and $B$ is an $n \times p$ matrix, then each column of $AB$ is a linear combination of the columns of $A$.
   b) If $A$ and $B$ are $n \times n$ and both are invertible, then the inverse of $AB$ is $A^{-1}B^{-1}$.
   c) If $A^T$ is not invertible, then $A$ is not invertible.
   d) If $A$ is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each $b$ in $\mathbb{R}^n$, then the solution is unique for each $b$ in $\mathbb{R}^n$.
   e) If $A$ and $B$ are invertible $n \times n$ matrices, then $A + B$ is invertible and $(A + B)^{-1} = A^{-1} + B^{-1}$.
   f) If $A$ and $B$ are $n \times n$ matrices and $ABx = 0$ has a unique solution, then $Ax = 0$ has a unique solution.

5. Suppose $A$ is an invertible $3 \times 3$ matrix and
   \[
   A^{-1} e_1 = \begin{pmatrix}
   4 \\
   1 \\
   0
   \end{pmatrix}, \quad A^{-1} e_2 = \begin{pmatrix}
   3 \\
   2 \\
   0
   \end{pmatrix}, \quad A^{-1} e_3 = \begin{pmatrix}
   0 \\
   0 \\
   1
   \end{pmatrix}.
   \]
   Find $A$. 

Math 1553 Worksheet §2.1, 2.2, 2.3