Office Hours (for now) Monday 1-2 and Wed 3-4 (today!)
If you want me to see your Piazza post, post to our group only
MyMathLab is not required required
Probably we will switch from T-Square to Canvas
Recitation Friday - no quiz this week
Eclipse brings learning opportunities to Georgia’s college campuses

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Georgia Tech students, faculty and visitors peer into the sky at the height of Monday’s solar eclipse. ERIC STIGUS / ESTIGUS@AJC.COM
Background

What is $\mathbb{R}^n$?
\( \mathbb{R} \) denotes the set of all real numbers

Geometrically, this is the \textit{number line}.

![Number line diagram]

\textbf{Definition}

\( \mathbb{R}^n = \) all ordered \( n \)-tuples of real numbers \((x_1, x_2, x_3, \ldots, x_n)\).

\textbf{Example}

For \( n = 1 \) we have \( \mathbb{R}^1 = \mathbb{R} \).
Example
When $n = 2$, we can think of $\mathbb{R}^2$ as the plane.

We can use the elements of $\mathbb{R}^2$ to *label* points on the plane, but $\mathbb{R}^2$ is not defined to be the plane!
Example
When \( n = 3 \), we can think of \( \mathbb{R}^3 \) as the *space* we (appear to) live in.

Again, we can use the elements of \( \mathbb{R}^3 \) to *label* points in space, but \( \mathbb{R}^3 \) is not defined to be space!
Example

We can think of the space of all colors as (a subset of) $\mathbb{R}^3$:

$$\text{all colors } (r, g, b) \subset \mathbb{R}^3.$$
So what is $\mathbb{R}^4$? or $\mathbb{R}^5$? or $\mathbb{R}^n$?

...go back to the definition: ordered $n$-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They’re still “geometric” spaces, in the sense that our intuition for $\mathbb{R}^2$ and $\mathbb{R}^3$ sometimes extends to $\mathbb{R}^n$, but they’re harder to visualize.
Last time we could have used $\mathbb{R}^4$ to label the amount of traffic \((x, y, z, w)\) passing through four streets.

We’ll make definitions and state theorems that apply to any $\mathbb{R}^n$, but we’ll only draw pictures in $\mathbb{R}^2$ and $\mathbb{R}^3$. 
This is a $21 \times 21$ QR code. We can also think of this as an element of $\mathbb{R}^n$.

How? Which $n$?

What about a greyscale image?
Background

Solving equations
Solving equations

What does it mean to solve an equation?

Example
\[2x = 10\]

Example
\[x^2 = 9\]

Example
\[x + y = 1\]
What does the solution set of a linear equation look like?

\[ x + y = 1 \implies \text{a line in the plane: } y = 1 - x \]

This is called the **implicit equation** of the line.
One Linear Equation

What does the solution set of a linear equation look like?

\[ x + y = 1 \quad \rightarrow \quad \text{a line in the plane: } y = 1 - x \]

This is called the **implicit equation** of the line.

We can write the same line in **parametric form**:

\[ (x, y) = (t, 1 - t) \quad t \text{ in } \mathbb{R}. \]

**Aside**

What is a line, anyway?
What does the solution set of a linear equation look like?

\[ x + y + z = 1 \]
What does the solution set of a linear equation look like?

\[ x + y + z = 1 \] is a plane in space:
This is the \textbf{implicit equation} of the plane.
What does the solution set of a linear equation look like?

\[ x + y + z = 1 \quad \rightarrow \text{a plane in space:} \]

This is the **implicit equation** of the plane.

This plane also has a parametric form:

\[(x, y, z) = (t, w, 1 - t - w) \quad t, w \text{ in } \mathbb{R}.

Note you need *two* parameters \( t \) and \( w \).

**Aside**

What is a plane?
What does the solution set of a linear equation look like?

\[ x + y + z + w = 1 \]
What does the solution set of a linear equation look like?

\[ x + y + z + w = 1 \rightarrow \text{“3-plane” in “4-space”...} \]
Is the plane from the previous example equal to $\mathbb{R}^2$?

A. Yes  
B. No

No! Every point on this plane is in $\mathbb{R}^3$: that means it has three coordinates. For instance, $(1, 0, 0)$. Every point in $\mathbb{R}^2$ has two coordinates. They're different planes.
What does the solution set of a system of more than one linear equation look like?

Example

\[ x - 3y = -3 \]
\[ 2x + y = 8 \]

What if there are more variables? More equations?
Kinds of Solution Sets

In what other ways can two lines intersect?

\[ x - 3y = -3 \]
\[ x - 3y = 3 \]

has no solution: the lines are parallel.

A system of equations with no solutions is called inconsistent.
In what other ways can two lines intersect?

\[ x - 3y = -3 \]
\[ x - 3y = 3 \]

A system of equations with no solutions is called **inconsistent**.
Kinds of Solution Sets

In what other ways can two lines intersect?

\[
x - 3y = -3 \\
2x - 6y = -6
\]

has infinitely many solutions: they are the same line.
Kinds of Solution Sets

In what other ways can two lines intersect?

\[ x - 3y = -3 \]
\[ 2x - 6y = -6 \]
Is it possible for two planes in $\mathbb{R}^4$ to intersect in one point?

A. Yes
B. No
We saw that there are three ways that two lines can intersect in \( \mathbb{R}^2 \): the intersection be be empty or a point or a line.

**Question.** In how many different ways can three lines intersect in the plane?

**Question.** In how many different ways can three planes intersect in space?