Announcements

Wednesday, August 23

- Office Hours (for now) Monday 1-2 and Wed 3-4 (today!)
- If you want me to see your Piazza post, post to our group only
- MyMathLab is not required required
- Probably we will switch from T-Square to Canvas
- Recitation Friday - no quiz this week

Eclipse brings learning opportunities to Georgia's college campuses

Eric Stirgus
5:34 p.m Monday, Aug. 21, 2017 Filed in AJC Homepage

Georgia Tech students, faculty and visitors peer into the sky at the height of Monday's solar eclipse. ERIC STIRGUS / ESTIRGUS@AJC.COM

MOST READ

1. Dunwoody day care murder: Andrea Sneiderman is free from parole

2. This is the healthiest part of the avocado

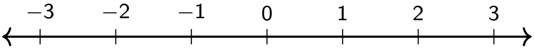
Background

What is $\mathbb{R}^n$?

$\mathbb{R}^n$

$\mathbb{R} =$ denotes the set of all real numbers

Geometrically, this is the *number line*.



Definition

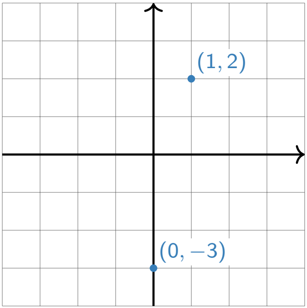
$$\mathbb{R}^n = \text{all ordered } n\text{-tuples of real numbers } (x_1, x_2, x_3, \ldots, x_n).$$

Example

For $n = 1$ we have $\mathbb{R}^1 = \mathbb{R}$.

Example

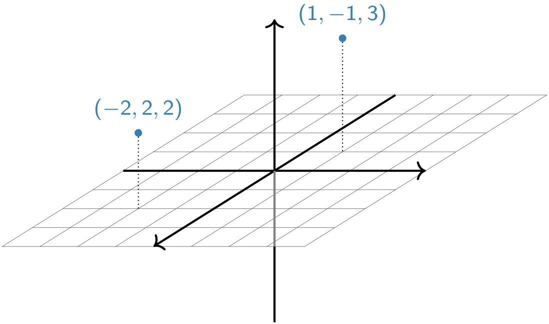
When $n = 2$, we can think of $\mathbb{R}^2$ as the *plane*.



We can use the elements of $\mathbb{R}^2$ to *label* points on the plane, but $\mathbb{R}^2$ is not defined to be the plane!

Example

When $n = 3$, we can think of $\mathbb{R}^3$ as the *space* we (appear to) live in.

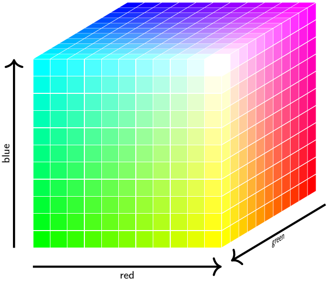


Again, we can use the elements of $\mathbb{R}^3$ to *label* points in space, but $\mathbb{R}^3$ is not defined to be space!

Example

We can think of the space of all *colors* as (a subset of) $\mathbb{R}^3$:

$$\text{all colors } (r, g, b) \subset \mathbb{R}^3.$$



$\mathbb{R}^n$

So what is $\mathbb{R}^4$? or $\mathbb{R}^5$? or $\mathbb{R}^n$?

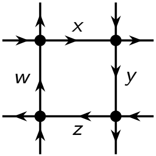
...go back to the *definition*: ordered n-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They're still "geometric" spaces, in the sense that our intuition for $\mathbb{R}^2$ and $\mathbb{R}^3$ sometimes extends to $\mathbb{R}^n$, but they're harder to visualize.

$\mathbb{R}^n$

Last time we could have used $\mathbb{R}^4$ to label the amount of traffic (x, y, z, w) passing through four streets.



We'll make definitions and state theorems that apply to any $\mathbb{R}^n$, but we'll only draw pictures in $\mathbb{R}^2$ and $\mathbb{R}^3$.

$\mathbb{R}^n$
and QR codes

This is a 21×21 QR code. We can also think of this as an element of $\mathbb{R}^n$.

How? Which n?

What about a greyscale image?

Background

Solving equations

Solving equations

What does it mean to solve an equation?

Example

$2x = 10$

Example

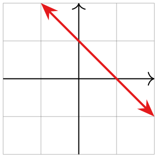
$x^2 = 9$

Example

$x + y = 1$

One Linear Equation

What does the solution set of a linear equation look like?



$x + y = 1 \rightsquigarrow$ a line in the plane: $y = 1 - x$
This is called the **implicit equation** of the line.

One Linear Equation

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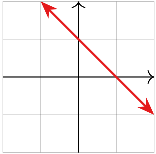

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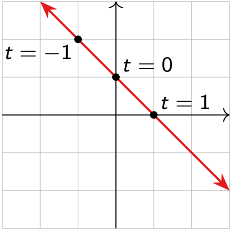


We can write the same line in **parametric form**:

$$(x, y) = (t, 1 - t) \qquad t \text{ in } \mathbb{R}.$$

Aside

What is a line, anyway?

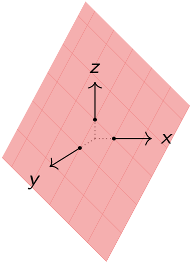
One Linear Equation

What does the solution set of a linear equation look like?

$x + y + z = 1$

One Linear Equation

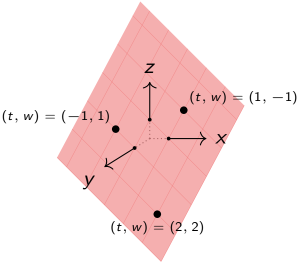
What does the solution set of a linear equation look like?



$x + y + z = 1 \rightsquigarrow$ a plane in space:
This is the **implicit equation** of the plane.

What does the solution set of a linear equation look like?

$x + y + z = 1 \rightsquigarrow$ a plane in space:
This is the **implicit equation** of the plane.



This plane also has a parametric form:

$$(x, y, z) = (t,\ w,\ 1 - t - w) \qquad t, w \text{ in } \mathbb{R}.$$

Note you need *two* parameters t and w.

Aside
What is a plane?

One Linear Equation
Continued

What does the solution set of a linear equation look like?

$x + y + z + w = 1$

One Linear Equation
Continued

What does the solution set of a linear equation look like?

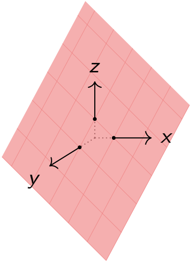
$x + y + z + w = 1 \rightsquigarrow$ a "3-plane" in "4-space"...

Poll

Poll

Is the plane from the previous example equal to $\mathbb{R}^2$?

A. Yes **B.** No



Systems of Linear Equations

What does the solution set of a *system* of more than one linear equation look like?

Example

$$x - 3y = -3$$
$$2x + y = 8$$

What if there are more variables? More equations?

Kinds of Solution Sets

In what other ways can two lines intersect?

$$x - 3y = -3$$
$$x - 3y = 3$$

Kinds of Solution Sets

In what other ways can two lines intersect?

$$x - 3y = -3$$
$$x - 3y = 3$$

A system of equations with no solutions is called **inconsistent**.

Kinds of Solution Sets

In what other ways can two lines intersect?

Kinds of Solution Sets

In what other ways can two lines intersect?

$$\begin{aligned} x - 3y &= -3 \\ 2x - 6y &= -6 \end{aligned}$$

Poll

Poll

Is it possible for two planes in $\mathbb{R}^4$ to intersect in one point?

A. Yes

B. No

A fun puzzle

We saw that there are three ways that two lines can intersect in $\mathbb{R}^2$: the intersection be be empty or a point or a line.

Question. In how many different ways can three lines intersect in the plane?

Question. In how many different ways can three planes intersect in space?