Announcements: August 30

- Office Hours today 3-4, Skiles 234
- Qianli Office Hours today 1-2, Skiles 153
- Kemi Office Hours Thursday 9:30-10:30, Skiles 230
- Martin's Office Hours Friday 2-3, Skiles 230
- WebWorK due Friday (only this week)
- Quiz in recitation on Friday (covers material up to Monday's class)
- Join our Piazza group: 1553 E1 through E5
- Set up your device so that you can quickly access Piazza during class

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Section 1.2

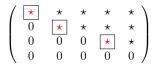
Row Reduction and Echelon Forms

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Row Reduction and Echelon Forms

A matrix is in row echelon form if

- 1. all zero rows are at the bottom,
- 2. each leading (nonzero) entry of a row is to the right of the leading entry of the row above, and
- 3. below a leading entry of a row all entries are zero.



This system is easy to solve using back substitution.

The pivot positions are the leading (nonzero) entries in each row.

Reduced Row Echelon Form

A system is in reduced row echelon form if also:

- 4. the leading entry in each nonzero row is 1
- 5. each leading entry of a row is the only nonzero entry in its column For example:

$$\left(\begin{array}{rrrrr} 1 & 0 & \star & 0 & \star \\ 0 & 1 & \star & 0 & \star \\ 0 & 0 & 0 & 1 & \star \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

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This system is even easier to solve.

Can every matrix be put in reduced row echelon form?

Reduced Row Echelon Form

Poll Which are in reduced row echelon form? $\left(\begin{array}{rrr}1&0\\0&2\end{array}\right)\quad\left(\begin{array}{rrr}0&0&0\\0&0&0\end{array}\right)$ $\left(\begin{array}{c}1\\0\end{array}\right) \quad \left(\begin{array}{cccc}0&1&0&0\end{array}\right) \quad \left(\begin{array}{ccccc}0&1&8&0\end{array}\right)$ $\left(\begin{array}{rrr}1 & 17 & 0\\0 & 0 & 1\end{array}\right)$

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REF:

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Row Reduction

Theorem. Each matrix is row equivalent to one and only one matrix in reduced row echelon form.

We'll give an algorithm. That shows a matrix is equivalent to at least one matrix in reduced row echelon form.

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Row Reduction Algorithm

To find row echelon form:

- Step 1 Swap rows so a leftmost nonzero entry is in 1st row (if needed)
- Step 2 Scale 1st row so that its leading entry is equal to 1
- Step 3 Use row replacement so all entries below this 1 (or, pivot) are 0

Then cover the first row and repeat the three steps.

To then find reduced row echelon form:

• Use row replacement so that all entries above the pivots are 0.

Examples.

$$\begin{pmatrix} 0 & 7 & -4 & | & 2 \\ 2 & 4 & 6 & | & 12 \\ 3 & 1 & -1 & | & -2 \end{pmatrix} \qquad \qquad \begin{pmatrix} 1 & 2 & 3 & | & 9 \\ 2 & -1 & 1 & | & 8 \\ 3 & 0 & -1 & | & 3 \end{pmatrix}$$

Solutions of Linear Systems

We want to go from reduced row echelon forms to solutions of linear systems.

Solve the linear system associated to:

$$\left(\begin{array}{cc|c}1 & 0 & 5\\0 & 1 & 2\end{array}\right)$$

What are the solutions?

Solutions of Linear Systems: Consistency

Solve the linear system associated to:

$$\left(\begin{array}{rrrr|r} 1 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$



Solutions of Linear Systems: Free Variables I

We want to go from reduced row echelon forms to solutions of linear systems.

Solve the linear system associated to:

$$\left(\begin{array}{rrrr|r} 1 & 0 & 5 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right)$$

This represents two equations:

$$x_1 + 5x_3 = 0 x_2 + 2x_3 = 1$$

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Solutions of Linear Systems: Free Variables II

Solve the linear system associated to:

$$\left(\begin{array}{rrrrr} 1 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$



Summary

There are *three possibilities* for the reduced row echelon form of the augmented matrix of a linear system.

1. The last column is a pivot column.

 \rightsquigarrow the system is *inconsistent*.

$$\begin{pmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 1 \end{pmatrix}$$

2. Every column except the last column is a pivot column. → the system has a *unique solution*.

$$\begin{pmatrix} 1 & 0 & 0 & \star \\ 0 & 1 & 0 & \star \\ 0 & 0 & 1 & \star \end{pmatrix}$$

The last column is not a pivot column, and some other column isn't either.

 → the system has *infinitely many* solutions; free variables correspond to columns without pivots.

$$\begin{pmatrix} 1 & \star & 0 & \star & | \\ 0 & 0 & 1 & \star & | \\ \end{pmatrix}$$

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Example with a parameter

For which values of h does the following system have a solution? For which values of h does it have a unique solution?

$$\begin{aligned} x + y &= 1\\ 2x + 2y &= h \end{aligned}$$

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A linear system has 4 variables and 3 equations. What are the possible solution sets?

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- 1. nothing
- 2. point
- 3. line

Poll

- 4. plane
- 5. 3-dimensional plane
- 6. 4-dimensional plane