Announcements: Sep 13

- Access your grades on Canvas
- During polls make sure to Save response
- Upcoming Office Hours
  - Me: today moved to 12-1, Skiles 234
  - Qianli: today 1-2, Clough 280
  - Arjun: today, 2:30-3:30, Skiles 230
  - Kemi: Thursday 9:30-10:30, Skiles 230
  - Martin: Friday 2-3, Skiles 230
- WebWorK due tonight
- Quiz in recitation on Friday (covers material from last week, Sec 1.3)

Section 1.4

The Matrix Equation $Ax = b$

Multiplying Matrices

$$\text{row} \times \text{column} : \begin{pmatrix} a_1 & \cdots & a_n \end{pmatrix} \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} =$$

$$\text{matrix} \times \text{column} : \begin{pmatrix} r_1 \\ \vdots \\ r_n \end{pmatrix} \begin{pmatrix} b \end{pmatrix} =$$

OR

$$\text{matrix} \times \text{column} : \begin{pmatrix} c_1 & \cdots & c_n \end{pmatrix} \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} =$$

Example:

$$\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} =$$

Linear Systems vs Augmented Matrices vs Matrix Equations vs Vector Equations

Matrix Equations vs Vector Equations

Say $u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

Write $3u - 5v + 7w = 0$ as a matrix equation.

Solutions to Linear Systems vs Spans

Fact. $Ax = b$ has a solution $\iff$ b is in the span of columns of A.

Why?

Solutions to Linear Systems vs Spans

Fact. $Ax = b$ has a solution $\iff$ b is in the span of columns of A.

Example:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} x = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

Is a given vector in the span?

Which of the following vectors are in the span of

$$(2, -1, 1), (1, 0, -1)?$$

- $(0, 2, 2)$
- $(3, -1, 0)$

Which of the following vectors are in the span of

$$(2, 3, 1, 4, 0), (3, 4, -1, 3, 5), (1, -1, 2, 4, 3)?$$

- $(3, 6, -5, -2, -7)$
- $(6, 19, -3, 4, -12)$

Is a given vector in the span?

Poll

Which of the following true statements can be checked without row reduction?

1. $(0, 1, 2)$ is in the span of $(3, 3, 4)$, $(0, 10, 20)$, $(0, -1, -2)$
2. $(0, 1, 2)$ is in the span of $(3, 3, 4)$, $(0, 5, 7)$, $(0, 6, 8)$
3. $(0, 1, 2)$ is in the span of $(3, 3, 4)$, $(0, 1, 0)$, $(0, 0, \sqrt{2})$
4. $(0, 1, 2)$ is in the span of $(5, 7, 0)$, $(6, 8, 0)$, $(3, 3, 4)$

Pivots vs Solutions

Theorem. Let A be an $m \times n$ matrix. The following are equivalent.

1. $Ax = b$ has a solution for all b
2. The span of the columns of A is...
3. A has a pivot in each...

Why?

Properties of the Matrix Product Ax

$c =$ real number, $u, v =$ vectors,

- $A(u + v) =$
- $A(cv) =$

Application. If u and v are solutions to $Ax = 0$ then so is every element of $\mathrm{Span}\{u + v\}$.

Solutions to $Ax = b$

Poll

If $b \neq 0$ then the set of solutions to $Ax = b$ is

1. always a span
2. sometimes a span
3. never a span