Announcements: Sep 6

- Access your grades on Canvas
- Office Hours today 3-4, Skiles 234
- Qianli's Office Hours today 1-2, Clough 280
- Arjun's Office Hours today, 2:30-3:30, Skiles 230
- Kemi's Office Hours Thursday 9:30-10:30, Skiles 230
- Martin's Office Hours Friday 2-3, Skiles 230
- WebWorK due tonight
- Quiz in recitation on Friday (covers material from last week)
- Minor changes to syllabus topics coming

Section 1.3

Vector Equations

Vectors

A *vector* is a matrix with one row or one column.

A length n vector can be drawn as a point or arrow in $\mathbb{R}^n$.

Adding vectors / parallelogram rule ▸ Demo

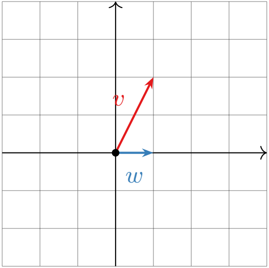
Scaling vectors ▸ Demo

Linear Combinations

A linear combination of the vectors $v_1, \ldots, v_k$ is any vector

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k$$

where $c_1, \ldots, c_k$ are real numbers.



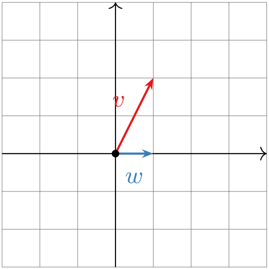
Let $v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

What are some linear combinations of v and w?

Poll

Is there a vector in $\mathbb{R}^2$ that is not a linear combination of v and w?

- true
- false



Linear Combinations

What are some linear combinations of $(1, 1)$?

What are some linear combinations of $(1, 1)$ and $(2, 2)$?

What are some linear combinations of $(0, 0)$?

Linear Combinations

Is $\begin{pmatrix} 8 \\ 16 \\ 3 \end{pmatrix}$ a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$?

Linear Combinations

So: Asking if b is a linear combination of $v_1, \ldots, v_k$ is the same as asking if the system of linear equations corresponding to the augmented matrix

$$\left(\begin{array}{cccc|c} | & | & & | & | \\ v_1 & v_2 & \cdots & v_p & b \\ | & | & & | & | \end{array}\right),$$

is consistent.

Span

$$\begin{aligned}\mathrm{Span}\{v_1, v_2, \ldots, v_k\} &= \{c_1v_1 + c_2v_2 + \cdots c_kv_k \mid c_i \text{ in } \mathbb{R}\} \leftarrow \text{(set builder notation)} \\ &= \text{the set of all linear combinations of vectors } v_1, v_2, \ldots, v_k \\ &= \text{plane through the origin and } v_1, v_2, \ldots, v_k.\end{aligned}$$

Four ways of saying the same thing:

- b is in Span$\{v_1, v_2, \ldots, v_k\}$
- b is a linear combination of $v_1, \ldots, v_k$
- the vector equation $c_1v_1 + \cdots + c_kv_k = b$ has a solution
- the system of linear equations corresponding to

$$\left(\begin{array}{cccc|c} | & | & & | & | \\ v_1 & v_2 & \cdots & v_p & b \\ | & | & & | & | \end{array}\right),$$

  is consistent.

Demo Demo

Application

Consider the production costs:

	Materials	Labor	Overhead
Widget	\$1	\$2	\$3
Gadget	\$4	\$5	\$6

Q. What are possible expenditures on materials, labor, and overhead?

Q. If we have a budget of \$11 for materials, \$16 for labor, and \$20 for overhead, can we spend our entire budget by making widgets and gadgets?

Poll

How many vectors are in the span of $(0, 0, 0)$.

- zero
- one
- infinitely many

Poll

If u is a linear combination of $v_1, v_2, \cdots v_k$, then there is only one way to write u as a linear combination of $v_1, v_2, \cdots v_k$ (that is, there is only one choice for $c_1, c_2, \ldots, c_k$).

- true
- false