Announcements: Sep 18

- Midterm 1 on Friday
- WebWork due tonight
- Review Sessions
  - Martin Thursday 6-8 Clough 125
  - Arjun Thursday 8-9:30 Skiles 202
- Upcoming Office Hours
  - Me: Today 11:15-12:15, Skiles 234 (unusual time)
  - Qianli: Wednesday 1-2, Clough 280
  - Arjun: Wednesday, 2:30-3:30, Skiles 230
  - Kemi: Thursday 9:30-10:30, Skiles 230
  - Bharat: Thursday 1:45-2:45, Skiles 230 (unusual day)
  - Martin: Friday 2-3, Skiles 230

Learning goals

Section 1.7

- Understand what is means for a set of vectors to be linearly independent
- Understand how to check if a set of vectors is linearly independent

Sections 1.8-1.9

- Learn to think of matrices as functions
- Understand what certain specific matrices do to $\mathbb{R}^n$

Section 1.7

Linear Independence

Linear Independence

Basic question: What is the smallest number of vectors needed in the parametric solution to a linear system?
A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

has only the trivial solution. It is linearly dependent otherwise.

So, linearly dependent means there are $c_1, c_2, \ldots, c_k$ not all zero so that

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

This is the *linear dependence* relation.

Linear Independence

A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

has only the trivial solution.

Fact. The cols of A are linearly independent $\Leftrightarrow$ $Ax = 0$...
$\Leftrightarrow$ A has a pivot in each...

Why?

Linear Independence

Is $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

Linear Independence

Is $\left\{ \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

Linear Independence

One vector

When is $\{v\}$ is linearly dependent?

Linear Independence

Two vectors

When is $\{v_1, v_2\}$ is linearly dependent?

Linear Independence

Any number of vectors

When is the set $\{v_1, v_2, \ldots, v_k\}$ linearly dependent?

Fact. The set $\{v_1, v_2, \ldots, v_k\}$ is linearly dependent if and only if some v_i lies in the span of $v_1, \ldots, v_{i-1}$.

Span and Linear Independence

Is $\left\{ \begin{pmatrix} 5 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

Fact 1. Say $v_1, \ldots, v_k$ are in $\mathbb{R}^n$. If $k > n$, then $\{v_1, \ldots, v_k\}$ is

Fact 2. If one of $v_1, \ldots, v_k$ is 0, then $\{v_1, \ldots, v_k\}$ is

Sections 1.8-1.9

Linear Transformations

From matrices to functions

Let A be an $m \times n$ matrix.

We define a function

$$T_A : \mathbb{R}^n \to \mathbb{R}^m$$
$$T_A(v) =$$

This is called a matrix transformation.

The domain of T_A is

The co-domain/target of T_A is

The range/image of T_A is

This gives us another point of view of $Ax = b$:

Example

Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$, $u = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $b = \begin{pmatrix} 7 \\ 5 \\ 7 \end{pmatrix}$.

What is $T_A(u)$?

Find v in $\mathbb{R}^2$ so that $T_A(v) = b$

Find c in $\mathbb{R}^3$ so there is no v with $T_A(v) = c$

Other ways to say this?

Dynamical systems

When A is a square matrix $(m = n)$ we can think of

$$T_A : \mathbb{R}^n \to \mathbb{R}^n$$

as doing something to $\mathbb{R}^n$.

Example. If

$$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

then

$$T_A \begin{pmatrix} x \\ y \end{pmatrix} =$$

What does T_A do to $\mathbb{R}^2$?

Examples in $\mathbb{R}^2$

What does T_A do to $\mathbb{R}^2$?

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

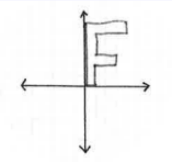
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

Clicker

Poll

What does $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ do to this letter F?



Examples in $\mathbb{R}^2$

What does T_A do to $\mathbb{R}^2$? *Hint: if you can't see it all at once, see what happens to the x- and y-axes.*

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Examples in $\mathbb{R}^3$

What does T_A do to $\mathbb{R}^3$?

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$