Announcements: Sep 18

- No quiz Friday
- WebWork due Friday
- Upcoming Office Hours
  - Me: Monday 1-2 and Wednesday 3-4, Skiles 234
  - Bharat: Tuesday 1:45-2:45, Skiles 230
  - Qianli: Wednesday 1-2, Clough 280
  - Arjun: Wednesday, 2:30-3:30, Skiles 230
  - Kemi: Thursday 9:30-10:30, Skiles 230
  - Martin: Friday 2-3, Skiles 230
- Midterm 2 October 20

Learning goals

Sections 1.8-1.9

- Learn to think of matrices as functions, called matrix transformations
- Understand what certain specific matrices do to $\mathbb{R}^n$
- Understand the definition of a linear transformation
- Linear transformations are the same as matrix transformations
- Determine when a linear transformation is one-to-one, onto

Sections 1.8-1.9

Linear Transformations

From matrices to functions

Let A be an $m \times n$ matrix.

We define a function

$$T : \mathbb{R}^n \to \mathbb{R}^m$$
$$T(v) =$$

This is called a matrix transformation.

The domain of T is

The co-domain of T is

The range of T is

This gives us another point of view of $Ax = b$

Example

Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$, $u = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $b = \begin{pmatrix} 7 \\ 5 \\ 7 \end{pmatrix}$.

What is $T(u)$?

Find v in $\mathbb{R}^2$ so that $T(v) = b$

Find c in $\mathbb{R}^3$ so there is no v with $T(v) = c$

Other ways to say this?

Dynamical systems

For a square matrix we can think of the associated matrix transformation

$$T : \mathbb{R}^n \to \mathbb{R}^n$$

as doing something to $\mathbb{R}^n$.

Example. The matrix transformation for

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

is

$$T\begin{pmatrix} x \\ y \end{pmatrix} =$$

What does T do to $\mathbb{R}^2$?

Examples in $\mathbb{R}^2$

What does each matrix do to $\mathbb{R}^2$?

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

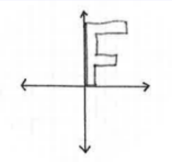
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

Clicker

Poll

What does $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ do to this letter F?



Examples in $\mathbb{R}^2$

What does each matrix do to $\mathbb{R}^2$?

Hint: if you can't see it all at once, see what happens to the x- and y-axes.

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Examples in $\mathbb{R}^3$

What does each matrix do to $\mathbb{R}^3$?

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$