Announcements: Oct 2

- WebWork 1.8 & 1.9 due Wednesday
- Quiz on 1.7, 1.8, & 1.9 on Friday
- Midsemester reflections on Canvas
- Upcoming Office Hours
  - Me: Monday 1-2 and Wednesday 3-4, Skiles 234
  - Bharat: Tuesday 1:45-2:45, Skiles 230
  - Qianli: Wednesday 1-2, Clough 280
  - Arjun: Wednesday, 2:30-3:30, Skiles 230
  - Kemi: Thursday 9:30-10:30, Skiles 230
  - Martin: Friday 2-3, Skiles 230
- Midterm 2 October 20

Learning goals

Sections 1.8-1.9

- How to find the matrix for a linear transformation
- Determine when a linear transformation is one-to-one, onto

Section 2.1

- How to multiply matrices
- Understand composition of linear transformations

Section 1.8-1.9

Linear Transformations II

Linear transformations are matrix transformations

Suppose $T : \mathbb{R}^2 \to \mathbb{R}^3$ is the function given by:

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y \\ x-y \end{pmatrix}$$

What is the standard matrix for T?

In fact, a function $\mathbb{R}^n \to \mathbb{R}^m$ is linear exactly when the coordinates are linear (linear combinations of the variables).

Linear transformations are matrix transformations

Find the standard matrix for the linear transformation of $\mathbb{R}^2$ that stretches by 2 in the x-direction and 3 in the y-direction, and then reflects over the line $y = x$.

One-to-one

$T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each b in $\mathbb{R}^m$ is the image of at most one v in $\mathbb{R}^n$.

Theorem. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with standard matrix A. Then the following are all equivalent:

- T is one-to-one
- the columns of A are linearly independent
- $Ax = 0$ has only the trivial solution
- A has a pivot in each column
- the range has dimension n

What can we say about the relative sizes of m and n if T is one-to-one?

Draw a picture of the range of a one-to-one mapping $\mathbb{R} \to \mathbb{R}^3$.

Onto

$T : \mathbb{R}^n \to \mathbb{R}^m$ is onto if the range of T is $\mathbb{R}^m$, that is, each b in $\mathbb{R}^m$ is the image of at least one v in $\mathbb{R}^m$.

Theorem. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with standard matrix A. Then the following are all equivalent:

- T is onto
- the columns of A span $\mathbb{R}^m$
- A has a pivot in each row
- $Ax = b$ is consistent for all b in $\mathbb{R}^m$.

What can we say about the relative sizes of m and n if T is onto?

Give an example of an onto mapping $\mathbb{R}^3 \to \mathbb{R}$.

Chapter 2

Matrix Algebra

Section 2.1

Matrix Operations

Terminology

Suppose A is an $m \times n$ matrix.

a_{ij} or A_{ij} is the ijth entry

a_{ii} are diagonal entries

diagonal matrix: all non-diagonal entries are 0

identity matrix: diagonal matrix with 1's on the diagonal

zero matrix: all entries are 0

the transpose of A is denoted A^T and has ij entry a_{ji}

Sums and Scalar Multiples

Same as for vectors: component-wise, so matrices must be same size to add.

$A + B =$

$(A + B) + C =$

$r(A + B) =$

$(r + s)A =$

$(rs)A =$

$A + 0 =$

Matrix Multiplication

If A is $m \times n$ and B is $n \times p$, then AB is $m \times p$ and

$$(AB)_{ij} = r_i \cdot b_j$$

where r_i is the ith row of A, and b_j is the jth column of B.

Or:

Matrix Multiplication and Linear Transformations

The composition $T \circ U$ means: do U then do T

Example. $T =$ projection to y-axis and $U =$ reflection about $y = x$ in $\mathbb{R}^2$

Question. What is the standard matrix for $T \circ U$?

Fact. The matrix for a composition of linear transformations is the product of the standard matrices.

Check by plugging in the e_i to $T \circ U$ and to the corresponding product.

Discussion Question

Are there nonzero matrices A and B with $AB = 0$?

1. Yes
2. No

Properties of Matrix Multiplication

- $A(BC) = (AB)C$
- $A(B + C) = AB + AC$
- $(B + C)A = BA + CA$
- $r(AB) = (rA)B = A(rB)$
- $(AB)^T = B^T A^T$
- $I_m A = A = A I_n$, where I_m is the $m \times m$ identity matrix.

Multiplication is associative because function composition is.

Warning!

- AB is not always equal to BA
- $AB = AC$ does not mean that $B = C$
- $AB = 0$ does not mean that A or B is 0