Announcements: Oct 30

- We need volunteers to do MatLab calculations in class today - get ready!
- WebWork 3.1, 3.2 due Wednesday
- Quiz 3.1, 3.2 Friday
- Midterm 3 on Friday Nov 17
- Upcoming Office Hours
  - Me: Monday 1-2 and Wednesday 3-4, Skiles 234
  - Bharat: Tuesday 1:45-2:45, Skiles 230
  - Qianli: Wednesday 1-2, Clough 280
  - Arjun: Wednesday, 2:30-3:30, Skiles 230
  - Kemi: Thursday 9:30-10:30, Skiles 230
  - Martin: Friday 2-3, Skiles 230

  Other sources of help:
  - Math Lab, Clough 280, Mon - Thu 12-6
  - Tutoring: http://www.successprograms.gatech.edu/tutoring

Where are we?

Remember:

> Almost every engineering problem, no matter how huge, can be reduced to linear algebra:
>
> $$Ax = b \quad \text{or}$$
> $$Ax = \lambda x$$

We have said most of what we are going to say about the first problem. We now begin in earnest on the second problem.

A Question from Biology

In a population of rabbits...

- half of the new born rabbits survive their first year
- of those, half survive their second year
- the maximum life span is three years
- rabbits produce 0, 6, 8 rabbits in their first, second, and third years

If I know the population one year, what is the population the next year?

Now choose some starting population vector $u = (f, s, t)$ and choose some number of years N. What is the new population after N years? $N + 1$ years?

Use a computer to find the actual numbers.

Eigenvectors and Eigenvalues

Suppose A is an $n \times n$ matrix and there is a $v \neq 0$ in $\mathbb{R}^n$ and λ in $\mathbb{R}$ so that

$$Av = \lambda v$$

then v is called an eigenvector for A, and λ is the corresponding eigenvalue.

eigen $=$ *characteristic*

So Av points in the same direction as v.

This the the most important definition in the course.

Eigenvectors and Eigenvalues

Examples

$$A = \begin{pmatrix} 0 & 6 & 8 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \end{pmatrix}, \quad v = \begin{pmatrix} 32 \\ 8 \\ 2 \end{pmatrix}, \quad \lambda = 2$$

$$A = \begin{pmatrix} 2 & 2 \\ -4 & 8 \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \lambda = 4$$

How do you check?

Eigenvectors and Eigenvalues

Confirming eigenvectors

Which of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ are eigenvectors of

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}?$$

What are the eigenvalues?

Eigenvectors and Eigenvalues

Confirming eigenvalues

Confirm that $\lambda = 3$ is an eigenvalue of $A = \begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix}$.

What is a general procedure for finding eigenvalues?

Eigenspaces

Let A be an $n \times n$ matrix. The set of eigenvectors for a given eigenvalue λ of A (plus the zero vector) is a subspace of $\mathbb{R}^n$ called the λ-eigenspace of A.

Why is this a subspace?

Fact. λ-eigenspace for $A = \mathrm{Nul}(A - \lambda I)$

Example. Find the eigenspaces for $\lambda = 2$ and $\lambda = -1$ and sketch.

$$\begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$$

Find a basis for the 2–eigenspace:

$$\begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$$

Eigenvalues

And invertibility

Fact. A invertible $\Leftrightarrow 0$ is not an eigenvalue of A

Why?

Eigenvalues

Triangular matrices

Fact. The eigenvalues of a triangular matrix are the diagonal entries.

Why?

Eigenvalues

Distinct eigenvalues

Fact. If $v_1 \dots v_k$ are distinct eigenvectors that correspond to distinct eigenvalues $\lambda_1, \dots \lambda_k$, then $\{v_1, \dots, v_k\}$ are linearly independent.

Why?

Eigenvalues geometrically

If v is an eigenvector of A then that means v and Av are scalar multiples, i.e. they lie on a line.

Suppose A corresponds to reflection about the line $y = -x$. What are the eigenvalues/eigenvectors of A?

Eigenvalues

Summary

- If $v \neq 0$ and $Av = \lambda v$ then λ is an eigenvector of A with eigenvalue λ
- The number λ is an eigenvalue of $A \leftrightarrow \det(A - \lambda I) = 0$
- The λ-eigenspace of A is the solution to $(A - \lambda I)x = 0$
- We can often see eigenvectors without doing calculations