Announcements: Dec 4

- CIOS open: 1 quiz for 80%
- WebWork 6.4 and 6.5 not due but on the final
- Final Exam here on Tuesday Dec 12 6:00-8:50pm
  Let me know of any conflicts ASAP
- Upcoming Office Hours
  - Me: Monday 1-2 and Wednesday 3-4, Skiles 234
  - Bharat: Tuesday 1:45-2:45, Skiles 230
  - Qianli: Wednesday 1-2, Clough 280
  - Arjun: Wednesday, 2:30-3:30, Skiles 230
  - Kemi: Thursday 9:30-10:30, Skiles 230
  - Martin: Friday 2-3, Skiles 230

  Other help:
  - Math Lab, Clough 280, Mon - Thu 12-6
  - Tutoring: http://www.successprograms.gatech.edu/tutoring
  - CAS Study Session Dec 6 Clough 144/152

Overview of the course

- Solving linear systems via row reduction
- The geometry of linear systems: Rank-Nullity Theorem
- Linear transformations
- Invertible Matrix Theorem
- Determinants
- Eigenvalues and Eigenvectors
- Orthogonality and the Method of Least Squares

Method of Least Squares

Find the least squares solution.

$$A = \begin{pmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 3 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$$

Least Squares Problems

Best fit line

Determine the least squares problem $Ax = b$ to find the best fit line for the points:

$$(2, 1), (5, 2), (0, 1)$$

Demo

Least Squares Problems

Best fit parabola

Determine the least squares problem $Ax = b$ to find the best parabola (quadratic function of x) for the points:

$$(0,0),(2,0),(3,0),(0,1)$$

Demo

Rank-Nullity

Suppose A is a 2×3 matrix and that the linear transformation $T(v) = Av$ is onto. Describe the solutions to $Ax = e_1$.

If the subspace of solutions to $Ax = 0$ has a basis consisting of three vectors and if $A$is a 5×7 matrix, what is the rank of A?

If possible, construct a 3×4 matrix with the dimension of the null space equal to 2 and the dimension of the column space equal to 1.

Construct a 4×3 matrix with rank 1.

Invertible Matrix Theorem

Suppose that A is an $n \times n$ matrix. Answer the following true/false questions.

If the rank of A is n then $Ax = e_1$ is consistent.

If $Ax = 0$ has a nontrivial solution then $T(v) = Av$ is onto.

If A is invertible then A^2 is invertible.

If A^2 is the zero matrix then A is not invertible.

Complex eigenvalues

Find the eigenvalues and eigenvectors of A. Find the rotation+scaling matrix to which A is similar.

$$A = \begin{pmatrix} -2 & 5 \\ -2 & 4 \end{pmatrix}$$

Linear transformations and orthogonality

Let

$$W = \mathrm{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

Find the matrix for orthogonal projection onto W. What are the eigenvalues and eigenspaces?

Old exam questions

Answer each of the following questions. You do not need to explain your answer.

The matrix $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is in reduced row echelon form.

TRUE FALSE

How many solutions are there for the linear system corresponding to the augmented matrix

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right)?$$

(a) 0
(b) 1
(c) infinitely many
(d) not enough information to determine

Old exam questions

Suppose A is an $m \times n$ matrix. Which of the following are equivalent to the statement that T_A is onto? Select all that apply.

(a) A has a pivot in each row
(b) the columns of A are linearly independent
(c) if $T_A(v) = T_A(w)$ then v must equal w
(d) for each input v to T_A there is exactly one output $T_A(v)$

Suppose that A is a 2×2 matrix and that the null space of A is the line $y = x$. Suppose also that b is a nonzero vector in $\mathbb{R}^2$. Which of the following are definitely not the set of solutions to $Ax = b$? Select all that apply.

(a) the line $y = x$
(b) the y–axis
(c) the line $y = x + 1$
(d) the zero vector

Old exam questions

Suppose A is an $n \times n$ matrix. Which of the following are equivalent to the statement that A is invertible? Select all that apply.

(a) the reduced row echelon form of A is the identity matrix
(b) A is similar to the identity matrix
(c) A is diagonalizable
(d) there is a matrix B with AB equal to to the identity

Old exam questions

Suppose that A is a 5×3 matrix and that the null space of A is a line. What is the range of T_A?

(a) a line in $\mathbb{R}^3$
(b) a plane in $\mathbb{R}^3$
(c) a line in $\mathbb{R}^5$
(d) a plane in $\mathbb{R}^5$

What is the area of the triangle in $\mathbb{R}^2$ with vertices $(1,1)$, $(5,6)$, and $(6,7)$?

Old exam questions

Suppose that A is an $n \times n$ matrix. Write down the definition of an eigenvector and eigenvalue.

Suppose that T_A is the linear transformation of $\mathbb{R}^3$ that orthogonally projects each vector onto the xy–plane. What are the eigenvalues of A?

Give an example of a 2×2 matrix that is diagonalizable but not invertible.

Old exam questions

Which of the following statements are equivalent to the statement that A is diagonalizable? Assume that A is an $n \times n$ matrix. Select all that apply.

(a) A is similar to a diagonal matrix

(b) A has at least one eigenvector for each eigenvalue

(c) for each eigenvalue λ of A, the dimension of the λ–eigenspace is equal to the algebraic multiplicity of λ

(d) A has n linearly independent eigenvectors for each eigenvalue