Name SOLUTIONS

Mathematics 1553

Midterm 2

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Section G1/Arjun G2/Talha G3/Athreya G4/Olivia G5/James (circle one!)

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1. Answer the following questions. No justification for your answer is required.

Suppose we have a set of 100 vectors in $\mathbb{R}^{99}$. Must it be true that the set is linearly dependent?

YES NO

Let A be an $m \times n$ matrix and let $T(v) = Av$ be the associated linear transformation. Suppose that T is not one-to-one. Must it be true that $Ax = 0$ has infinitely many solutions?

YES NO

Consider the function $T : \mathbb{R} \to \mathbb{R}$ given by $T(x) = x + 1$. Is T a linear transformation?

YES NO

Suppose that $\{u, v\}$ is a basis for a subspace V of $\mathbb{R}^3$. Must it be true that $\{u + v, v\}$ is a basis for V?

YES NO

2. Answer the following questions. No justification for your answer is required.

Complete the definition: *A set of vectors* $\{v_1, \ldots, v_k\}$ *in* $\mathbb{R}^n$ *is linearly independent if...*

the vector equation

$$c_1 v_1 + \cdots + c_k v_k = 0$$

has only the trivial solution.

Consider the plane $z = 1$ in $\mathbb{R}^3$. Which properties of a subspace are *failed* by V? Select all that apply.

(a) Zero vector: the zero vector is in V

(b) Closure under addition: if u and v are in V then $u + v$ is in V

(c) Closure under scalar multiplication: if u is in V and c is a scalar then cu is in V

(d) None of the above; V is a subspace

Write down a 2×2 matrix so that the null space and column space both equal the line $y = x$.

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$$

Write down a *nonzero* 2×2 matrix A so that $A \neq I$ and $A^2 = A$.

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

3. Consider the following matrix and its reduced row echelon form:

$$A = \begin{pmatrix} 2 & 4 & 1 & 11 & 0 \\ 3 & 6 & 1 & 16 & 0 \\ 7 & 14 & 3 & 38 & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 & 5 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Find a basis for Col(A).

$$\left\{ \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right\}$$

What is the dimension of Col(A)?

2

Find a basis for Nul(A).

$x_1 = -2x_2 - 5x_4$

$x_3 = -x_4$

x_2, x_4, x_5 free.

$$\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

What is the dimension of Nul(A)? 3

Is it possible for a 3×5 matrix to have the dimensions of its column space and null space be equal?

YES NO

4. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects over the line $y = -x$. What is the standard matrix for T?

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Consider the linear transformation $U : \mathbb{R}^3 \to \mathbb{R}^2$ given by the formula

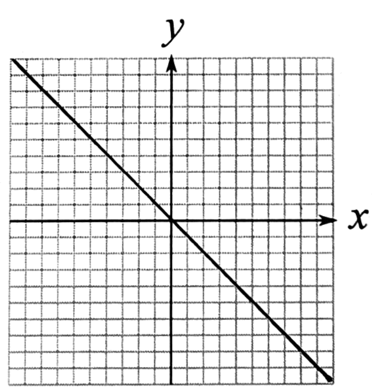
$$U \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ -x \end{pmatrix}$$

What is the standard matrix for U?

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

Is U one-to-one? YES NO

Draw the range of U.



What is the standard matrix for $T \circ U$?

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

5. Consider the set of vectors

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ h \\ 3 \end{pmatrix} \right\}$$

For which values of h is the set linearly dependent?

$$\begin{pmatrix} 1 & 2 & 5 \\ 1 & -1 & h \\ 1 & 1 & 3 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 5 \\ 1 & 1 & 3 \\ 1 & -1 & h \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 5 \\ 0 & +1 & +2 \\ 0 & -3 & h-5 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & h+1 \end{pmatrix} \qquad \boxed{h=-1}$$

6. Find a basis for the plane in $\mathbb{R}^3$ defined by the equation $x + 2y + z = 0$.

$$(1 \ \ 2 \ \ 1) \qquad \begin{aligned} x &= -2y - z \\ y &= y \\ z &= z \end{aligned}$$

$$\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$