Name SOLUTIONS

Mathematics 1553

Midterm 1

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Section G1/Arjun G2/Talha G3/Athreya G4/Olivia G5/James (circle one!)

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1. Answer the following questions. No justification for your answer is required.

Is the matrix $\left(\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right)$ in reduced row echelon form?

YES NO

Suppose that A is an 3×4 matrix and that b is the vector obtained by adding together the first two columns of A. Must it be true that $Ax = b$ is consistent?

YES NO

Suppose we have four variables x_1, x_2, x_3, and x_4. Is the set of solutions to $x_1 + x_2 = 0$ a 3-dimensional plane?

YES NO

Suppose A is a 4×3 matrix. Must it be true that the set of solutions to $Ax = 0$ is a span?

YES NO

2. Answer the following questions. No justification for your answer is required.

Complete the following definition: *A linear combination of the vectors $v_1, \ldots, v_k$ is...*

a vector $c_1 v_1 + \cdots + c_k v_k$

where $c_1, \ldots, c_k$ are scalars.

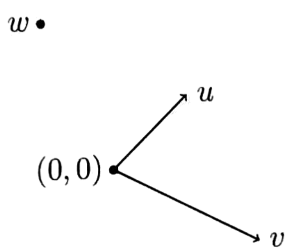
Write down a 2×3 matrix A so that $Ax = b$ is consistent for every choice of b in $\mathbb{R}^2$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Write down one vector in $\mathbb{R}^3$ that is not in the span of the vectors $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Circle the formula that best describes w in terms of u and v.



$u - v$ $\qquad$ $v - u$ $\qquad$ $-u - v$ $\qquad$ $2u - v$

3. Consider the matrices

$$A = \begin{pmatrix} 2 & -4 & 0 & 8 \\ 3 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \end{pmatrix} \text{ and } b = \begin{pmatrix} 10 \\ 3 \\ -1 \end{pmatrix}.$$

Find the reduced row echelon form of the augmented matrix $(A \,|\, b)$.

$$\rightsquigarrow \left(\begin{array}{cccc|c} 1 & -2 & 0 & 4 & 5 \\ 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & -1 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & -2 & 0 & 4 & 5 \\ 0 & 2 & 0 & -4 & -4 \\ 0 & -1 & 1 & 4 & 4 \end{array}\right)$$

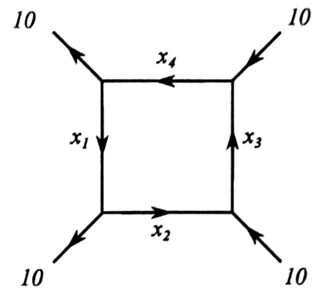
$$\rightsquigarrow \left(\begin{array}{cccc|c} 1 & -2 & 0 & 4 & 5 \\ 0 & 1 & 0 & -2 & -2 \\ 0 & -1 & 1 & 4 & 4 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 & -2 \\ 0 & 0 & 1 & 2 & 2 \end{array}\right)$$

Write the set of solutions to the matrix equation $Ax = b$ in vector parametric form.

$$x_1 = 1$$
$$x_2 = -2 + 2x_4$$
$$x_3 = 2 - 2x_4$$

$$\begin{pmatrix} 1 \\ -2 \\ 2 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 2 \\ -2 \\ 1 \end{pmatrix}$$

4. The following diagram indicates traffic flow in the town square (the numbers indicate the number of cars per minute on each section of road).



Write a system of linear equations in x_1, x_2, x_3, and x_4 describing the traffic flow around the square. Do not solve.

$$x_4 = x_1 + 10$$
$$x_3 + 10 = x_4$$
$$x_2 + 10 = x_3$$
$$x_1 = x_2 + 10$$

$$\begin{aligned} x_1 \qquad\qquad - x_4 &= -10 \\ x_3 - x_4 &= -10 \\ x_2 - x_3 \qquad &= -10 \\ x_1 - x_2 \qquad\qquad &= 10 \end{aligned}$$

Write the above system of linear equations as a vector equation. Do not solve.

$$x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} -10 \\ -10 \\ -10 \\ 10 \end{pmatrix}$$

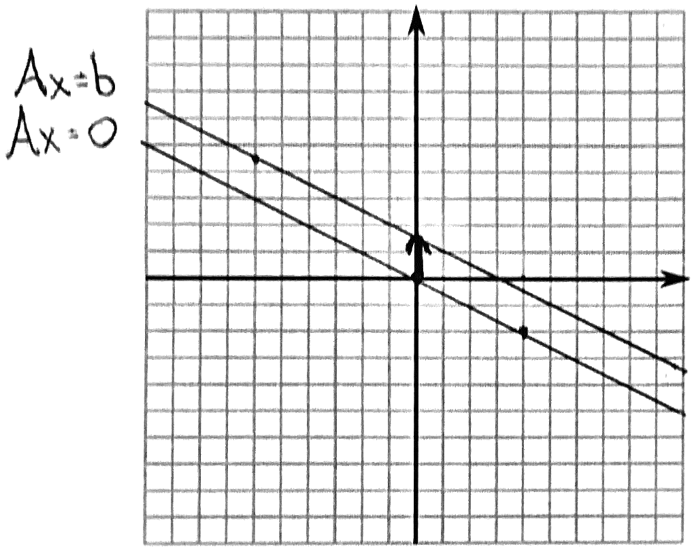
Write the above system of linear equations as a matrix equation. Do not solve.

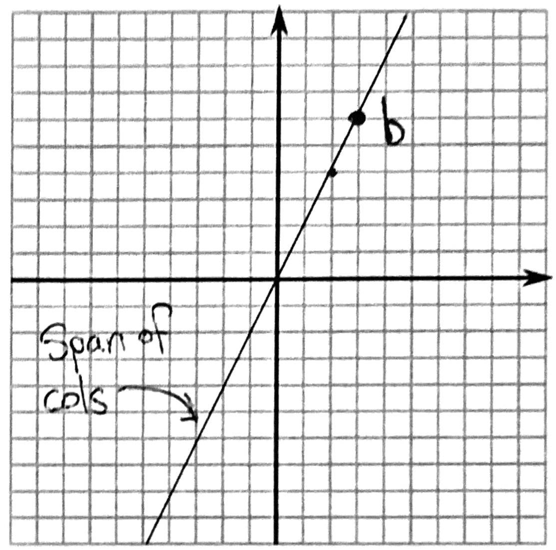
$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -10 \\ -10 \\ -10 \\ 10 \end{pmatrix}$$

5. Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \text{ and } b = \begin{pmatrix} 3 \\ 6 \end{pmatrix}.$$

On the *left-hand side* draw and label three things: the set of solutions to $Ax = 0$, the set of solutions to $Ax = b$, and one particular solution to $Ax = b$ as a vector/arrow. On the *right-hand side* draw and label two things: the span of the columns of A and b as a dot.





$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \qquad x + 2y = 0 \qquad X + 2y = 3$$

$$y = -\tfrac{1}{2}X \qquad y = -\tfrac{1}{2}x + \tfrac{3}{2}$$

For which values of h does the matrix equation $Ax = \begin{pmatrix} -1 \\ h \end{pmatrix}$ have a solution?

$h = -2$

Is there a different choice of b so that the set of solutions to $Ax = b$ is the line $y = x$? If so, write down such a b. If not, briefly explain why not.

No. $y = x$ is not parallel to $y = -\tfrac{1}{2}X$.