MATH 1553, FALL 2018
SAMPLE MIDTERM 1: THROUGH SECTION 3.4

Name	

Please **read all instructions** carefully before beginning.

- You have 50 minutes to complete this exam.
- There are no aids of any kind (calculators, notes, text, etc.) allowed.
- Please show your work unless specified otherwise. A correct answer without appropriate work may be given little or no credit.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

Problem 1.

[Parts a) through f) are worth 2 points each]

a) Compute: $\begin{pmatrix} 3 & 2 \\ -2 & 0 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} =$

The remaining problems are True or false. Circle **T** if the statement is **always** true, and circle **F** otherwise. You do not need to justify your answer.

b) **T** **F** The matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ is in reduced row echelon form.

c) **T** **F** If $Ax = b$ is consistent, then the equation $Ax = 5b$ is consistent.

d) **T** **F** If the augmented matrix corresponding to a linear system of equations has a pivot in every row, then the system is consistent.

e) **T** **F** If A is an $m \times n$ matrix and $Ax = 0$ has a unique solution, then $Ax = b$ is consistent for every b in $\mathbf{R}^m$.

f) **T** **F** The three vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ span $\mathbf{R}^3$.

Problem 2.

Parts (a) and (b) are 2 points each. Parts (c) and (d) are 3 points each.

a) If A is a 2×3 matrix with 2 pivots, then the set of solutions to $Ax = 0$ is a:

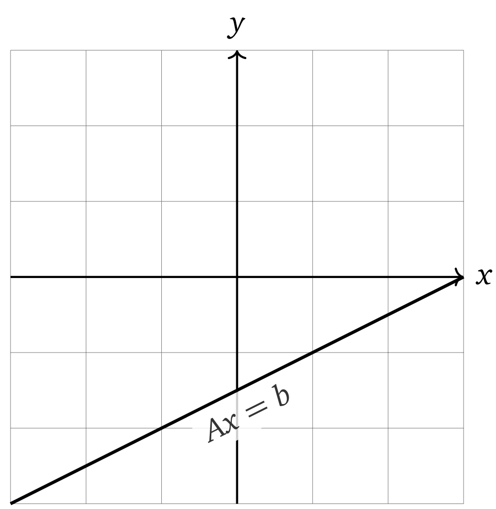
(circle one answer) **point** **line** **plane** **3-plane**

in:

(circle one answer) $\mathbf{R}$ $\mathbf{R}^2$ $\mathbf{R}^3$.

b) Write a vector equation which represents an inconsistent system of two linear equations in x_1 and x_2.

c) For some 2×2 matrix A and vector b in $\mathbf{R}^2$, the solution set of $Ax = b$ is drawn below. Draw the solution set of $Ax = 0$.



d) If b, v, w are vectors in $\mathbf{R}^3$ and $\text{Span}\{b, v, w\} = \mathbf{R}^3$, is it possible that b is in $\text{Span}\{v, w\}$? Justify your answer.

Problem 3. [10 points]

Johnny Rico believes that the secret to the universe can be found in the system of two linear equations in x and y given by

$$\begin{aligned} x - y &= h \\ 3x + hy &= 4 \end{aligned}$$

where h is a real number.

a) Find all values of h (if any) which make the system inconsistent. Briefly justify your answer.

b) Find all values of h (if any) which make the system have a unique solution. Briefly justify your answer.

Problem 4.

[11 points]

a) Find the parametric form of the general solution of the following system of equations. Clearly indicate which variables (if any) are free variables.

$$\begin{aligned} x_1 + 2x_2 + 2x_3 - x_4 &= 4 \\ 2x_1 + 4x_2 + x_3 - 2x_4 &= -1 \\ -x_1 - 2x_2 - x_3 + x_4 &= -1 \end{aligned}$$

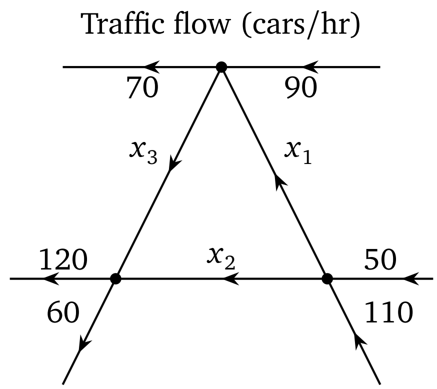
b) Write the set of solutions to

$$\begin{aligned} x_1 + 2x_2 + 2x_3 - x_4 &= 0 \\ 2x_1 + 4x_2 + x_3 - 2x_4 &= 0 \\ -x_1 - 2x_2 - x_3 + x_4 &= 0 \end{aligned}$$

in parametric vector form.

Problem 5. [7 points]

The diagram below represents traffic in a city.



a) Write a system of three linear equations whose solution would give the values of x_1, x_2, and x_3. Do not solve it.

b) Write the system of equations as a vector equation. Do not solve it.

[Scratch work]