Announcements August 21

- Mathematical autobiography due on Friday
- WeBWorK Warmup due Friday (not for a grade)
- My office hours today 2-3 and Friday 9-10 in Skiles 234
- Recitation on Friday: same time, different room, with TA
- Remember the laptop rules
Section 2.1

Solving systems of equations
Outline of Section 2.1

- Learn what it means to solve a system of linear equations
- Describe the solutions as points in $\mathbb{R}^n$
- Learn what it means for a system of linear equations to be inconsistent
Solving equations
Solving equations

What does it mean to solve an equation?

\[2x = 10\]

\[x + y = 1\]

\[x + y + z = 0\]

Find one solution to each. Can you find all of them?

A solution is a *list* of numbers. For example \((3, -4, 1)\).
Solving equations

What does it mean to solve a system of equations?

\[
\begin{align*}
x + y &= 2 \\
y &= 1
\end{align*}
\]

What about...

\[
\begin{align*}
x + y + z &= 3 \\
x + y - z &= 1 \\
x - y + z &= 1
\end{align*}
\]

Is \((1, 1, 1)\) a solution? Is \((2, 0, 1)\) a solution? What are all the solutions?

Soon, you will be able to see just by looking that there is exactly one solution.
$\mathbb{R}^n$
\( \mathbb{R} \) denotes the set of all real numbers

Geometrically, this is the *number line*.

\[ -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \]

\( \mathbb{R}^n \) = all ordered \( n \)-tuples (or lists) of real numbers \((x_1, x_2, x_3, \ldots, x_n)\)

Solutions to systems of equations are exactly points in \( \mathbb{R}^n \).
When $n = 2$, we can visualize $\mathbb{R}^2$ as the *plane*. 

![Diagram of $\mathbb{R}^2$ with points (1, 2) and (0, -3)](attachment:image.png)
When $n = 3$, we can visualize $\mathbb{R}^3$ as the *space* we (appear to) live in.
We can think of the space of all *colors* as (a subset of) $\mathbb{R}^3$:  

![Color cube diagram](image)
So what is $\mathbb{R}^4$? or $\mathbb{R}^5$? or $\mathbb{R}^n$?

...go back to the definition: ordered $n$-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They’re still “geometric” spaces, in the sense that our intuition for $\mathbb{R}^2$ and $\mathbb{R}^3$ sometimes extends to $\mathbb{R}^n$, but they’re harder to visualize.
Last time we could have used $\mathbb{R}^4$ to label the amount of traffic $(x, y, z, w)$ passing through four streets.

We'll make definitions and state theorems that apply to any $\mathbb{R}^n$, but we'll only draw pictures in $\mathbb{R}^2$ and $\mathbb{R}^3$. 
This is a $21 \times 21$ QR code. We can also think of this as an element of $\mathbb{R}^n$.

How? Which $n$?

What about a greyscale image?

This is a powerful idea: instead of thinking of a QR code as 441 pieces of information, we think of it as one piece of information.
Visualizing solutions: a preview
One Linear Equation

What does the solution set of a linear equation look like?

\[ x + y = 1 \implies \text{a line in the plane: } y = 1 - x \]
One Linear Equation

What does the solution set of a linear equation look like?

\[ x + y + z = 1 \]  \implies \text{a plane in space:}
What does the solution set of a linear equation look like?

\[x + y + z + w = 1 \rightarrow \text{a “3-plane” in “4-space”...} \]
Systems of Linear Equations

What does the solution set of a *system* of more than one linear equation look like?

\[
\begin{align*}
x - 3y &= -3 \\
2x + y &= 8
\end{align*}
\]

What are the other possibilities for two equations with two variables?

What if there are more variables? More equations?
Is the plane in $\mathbb{R}^3$ from the previous example equal to $\mathbb{R}^2$? What about the $xy$-plane in $\mathbb{R}^3$?

1. yes + yes
2. yes + no
3. no + yes
4. no + no

Answer. No + no. To describe any point in $\mathbb{R}^3$, you need three numbers, not a list of two numbers. The $xy$-plane in $\mathbb{R}^3$ is the set of all triples $(x, y, 0)$. So it's like $\mathbb{R}^2$, but it is not.
Poll

Is the plane in $\mathbb{R}^3$ from the previous example equal to $\mathbb{R}^2$? What about the $xy$-plane in $\mathbb{R}^3$?

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2. yes + no
3. no + yes
4. no + no

Answer. No + no. To describe any point in $\mathbb{R}^3$, you need three numbers, not a list of two numbers. The $xy$-plane in $\mathbb{R}^3$ is the set of all triples $(x, y, 0)$. So it’s like $\mathbb{R}^2$, but it is not.
Consistent versus Inconsistent

We say that a system of linear equations is consistent if it has a solution and inconsistent otherwise.

\[ x + y = 1 \]
\[ x + y = 2 \]

Why is this inconsistent?

What are other examples of inconsistent systems of linear equations?
Summary of Section 2.1

- A solution to a system of linear equations in \( n \) variables is a point in \( \mathbb{R}^n \).
- The set of all solutions to a single equation in \( n \) variables is an \((n - 1)\)-dimensional plane in \( \mathbb{R}^n \).
- The set of solutions to a system of \( m \) linear equations in \( n \) variables is the intersection of \( m \) of these \((n - 1)\)-dimensional planes in \( \mathbb{R}^n \).
- A system of equations with no solutions is said to be inconsistent.