Announcements: Sep 17

- Midterm 1 on Friday
- WeBWorK 3.3 and 3.4 due tonite!
- My office hours Wednesday 2:00-3:00 and Friday 9:30-10:30 in Skiles 234
- TA Office Hours
  - Arjun Wed 3-4 Skiles 230
  - Talha Tue/Thu 11-12 Clough 248
  - Athreya Tue 3-4 Skiles 230
  - Olivia Thu 3-4 Skiles 230
  - James Fri 12-1 Skiles 230
  - Jesse Wed 9:30-10:30 Skiles 230
  - Vajraang Thu 9:30-10:30 Skiles 230
  - Hamed Thu 11:15-12, 1:45-2:45, 3-4:15 Clough 280
- Math Lab Monday - Thursday 11:15-5:15 Clough 280
- PLUS Sessions
  - Tue/Thu 6-7 Clough 280
  - Mon/Wed 7-8 Clough 123
- Review Sessions
  - Talha Thu 7-8:30 Mason 1133 (doors close at 7!)
  - Arjun tba
- Supplemental problems and practice exams on master course web site

Section 3.5

Linear Independence

Section 1.7 Outline

- Understand what is means for a set of vectors to be linearly independent
- Understand how to check if a set of vectors is linearly independent

Linear Independence

Basic question: What is the smallest number of vectors needed in the parametric solution to a linear system?

A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

has only the trivial solution. It is linearly dependent otherwise.

So, linearly dependent means there are $c_1, c_2, \ldots, c_k$ not all zero so that

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

This is a *linear dependence* relation.

Linear Independence

A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

has only the trivial solution.

Fact. The cols of A are linearly independent
$\Leftrightarrow$ $Ax = 0$ has only the trivial solution.
$\Leftrightarrow$ A has a pivot in each column

Why?

Linear Independence

Is $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

Is $\left\{ \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

Linear Independence

When is $\{v\}$ is linearly dependent?

When is $\{v_1, v_2\}$ is linearly dependent?

When is the set $\{v_1, v_2, \ldots, v_k\}$ linearly dependent?

Fact. The set $\{v_1, v_2, \ldots, v_k\}$ is linearly dependent if and only if we can remove a vector from the set without changing the dimension of the span.

Fact. The set $\{v_1, v_2, \ldots, v_k\}$ is linearly dependent if and only if some v_i lies in the span of $v_1, \ldots, v_{i-1}$.

Span and Linear Independence

Is $\left\{ \begin{pmatrix} 5 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

Linear independence and free variables

Theorem. Let $v_1, \ldots, v_k$ be vectors in $\mathbb{R}^n$ and consider the vector equation

$$c_1 v_1 + \cdots + c_k v_k = 0.$$

The set of vectors corresponding to non-free variables are linearly independent.

So if we put the v_i as the columns of a matrix, the number of pivots is the dimension of the span. Actually, the columns of the *original matrix* corresponding to the pivots are linear independent.

Linear independence and coordinates

Theorem. If $v_1, \ldots, v_k$ are linearly independent vectors then we can write each element of

$$\text{Span}\{v_1, \ldots, v_k\}$$

in exactly one way as a linear combination of $v_1, \ldots, v_k$.

More on this later, when we get to bases.

Fact 1. Say $v_1, \ldots, v_k$ are in $\mathbb{R}^n$. If $k > n$, then $\{v_1, \ldots, v_k\}$ is linearly dependent.

Fact 2. If one of $v_1, \ldots, v_k$ is 0, then $\{v_1, \ldots, v_k\}$ is linearly dependent.

Summary of Section 3.5

- A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

has only the trivial solution. It is linearly dependent otherwise.

- The cols of A are linearly independent
  $\Leftrightarrow$ $Ax = 0$ has only the trivial solution.
  $\Leftrightarrow$ A has a pivot in each column
- The number of pivots of A equals the dimension of the span of the columns of A
- The set $\{v_1, v_2, \ldots, v_k\}$ is linearly dependent if and only if some v_i lies in the span of $v_1, \ldots, v_{i-1}$.