Announcements: Sep 19

- Midterm 2 Oct 19 in recitation
- No quiz Friday in recitation
- WeBWorK 3.5 and 3.6 due Wednesday
- My office hours Wed 2-3 and Friday 9:30-10:30 in Skiles 234
- TA Office Hours
  - Arjun Wed 3-4 Skiles 230
  - Talha Tue/Thu 11-12 Clough 248
  - Athreya Tue 3-4 Skiles 230
  - Olivia Thu 3-4 Skiles 230
  - James Fri 12-1 Skiles 230
  - Jesse Wed 9:30-10:30 Skiles 230
  - Vajraang Thu 9:30-10:30 Skiles 230
  - Hamed Thu 11:15-12, 1:45-2:45, 3-4:15 Clough 280
- Math Lab Monday-Thursday 11:15-5:15 Clough 280
- PLUS Sessions
  - Tue/Thu 6-7 Clough 280
  - Mon/Wed 7-8 Clough 123
- Supplemental problems and practice exams on master course web site

Section 3.7

Bases

Bases

V = subspace of $\mathbb{R}^n$

A basis for V is a set of vectors $\{v_1, v_2, \ldots, v_k\}$ such that

1. $V = \text{Span}\{v_1, \ldots, v_k\}$
2. $v_1, \ldots, v_k$ are linearly independent

Equivalently, a basis is a *minimal spanning set*, that is, a spanning set where if you remove any one of the vectors you no longer have a spanning set.

$\dim(V)$ = dimension of $V = k$ =the number of vectors in the basis

(What is the problem with this definition of dimension?)

Q. What is one basis for $\mathbb{R}^2$? $\mathbb{R}^n$? How many bases are there?

Bases for $\mathbb{R}^n$

What are all bases for $\mathbb{R}^n$?

Take a set of vectors $\{v_1, \ldots, v_k\}$. Make them the columns of a matrix.

For the vectors to be linearly independent we need a pivot in every column.

For the vectors to span $\mathbb{R}^n$ we need a pivot in every row.

Conclusion: $k = n$ and the matrix has n pivots.

Who cares about bases

A basis $\{v_1, \ldots, v_k\}$ for a subspace V of $\mathbb{R}^n$ is useful because:

Every vector v in V can be written in exactly one way:

$$v = c_1 v_1 + \cdots + c_k v_k$$

So a basis gives coordinates for V, like latitude and longitude. See Section 3.8.

Bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

Find bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

Find bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

In general:

- our usual parametric solution for $Ax = 0$ gives a basis for $\mathrm{Nul}(A)$
- the pivot columns of A form a basis for $\mathrm{Col}(A)$

Warning! Not the pivot columns of the reduced matrix.

What should you do if you are asked to find a basis for Span$\{v_1, \ldots, v_k\}$?

Bases for planes

Find a basis for the plane $2x + 3y + z = 0$ in $\mathbb{R}^3$.

Basis theorem

Basis Theorem

If V is a k-dimensional subspace of $\mathbb{R}^n$, then

- any k linearly independent vectors of V form a basis for V
- any k vectors that span V form a basis for V

In other words if a set has two of these three properties, it is a basis:

$$\text{spans } V, \text{ linearly independent}, \; k \text{ vectors}$$

We are skipping Section 3.8 this semester. But remember: the whole point of a basis is that it gives coordinates (like latitude and longitude) for a subspace. Every point has a unique address.

Section 3.7 Summary

- A basis for a subspace V is a set of vectors $\{v_1, v_2, \ldots, v_k\}$ such that
  1. $V = \text{Span}\{v_1, \ldots, v_k\}$
  2. $v_1, \ldots, v_k$ are linearly independent
- The number of vectors in a basis for a subspace is the dimension.
- Find a basis for $\text{Nul}(A)$ by solving $Ax = 0$ in vector parametric form
- Find a basis for $\text{Col}(A)$ by taking pivot columns of A (not reduced A)
- Basis Theorem. Suppose V is a k-dimensional subspace of $\mathbb{R}^n$. Then
  - Any k linearly independent vectors in V form a basis for V.
  - Any k vectors in V that span V form a basis.