Announcements: Sep 26

- Midterm 2 Oct 19 in recitation
- **No quiz** Friday in recitation
- WeBWorK 3.5 and 3.6 due Wednesday
- My office hours **Wed 2-3** and Friday 9:30-10:30 in Skiles 234
- TA Office Hours
  - Arjun Wed 3-4 Skiles 230
  - Talha Tue/Thu 11-12 Clough 248
  - Athreya Tue 3-4 Skiles 230
  - Olivia Thu 3-4 Skiles 230
  - James Fri 12-1 Skiles 230
  - Jesse Wed 9:30-10:30 Skiles 230
  - Vajraang Thu 9:30-10:30 Skiles 230
  - Hamed Thu 11:15-12, 1:45-2:45, 3-4:15 Clough 280
- Math Lab Monday-Thursday 11:15-5:15 Clough 280
- PLUS Sessions
  - Tue/Thu 6-7 Clough 280
  - Mon/Wed 7-8 Clough 123
- Supplemental problems and practice exams on master course web site
Sections 4.1

Matrix Transformations
Section 4.1 Outline

- Learn to think of matrices as functions, called matrix transformations
- Learn the associated terminology: domain, codomain, range
- Understand what certain matrices do to $\mathbb{R}^n$
From matrices to functions

Let $A$ be an $m \times n$ matrix.

We define a function

$$T : \mathbb{R}^n \to \mathbb{R}^m$$

$$T(v) = Av$$

This is called a matrix transformation.

The domain of $T$ is $\mathbb{R}^n$. The co-domain of $T$ is $\mathbb{R}^m$.

The range of $T$ is the set of outputs: $\text{Col}(A)$

This gives us another point of view of $Ax = b$
Example

Let \( A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \), \( u = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \), \( b = \begin{pmatrix} 7 \\ 5 \\ 7 \end{pmatrix} \).

What is \( T(u) \)?

Find \( v \) in \( \mathbb{R}^2 \) so that \( T(v) = b \)

Find a vector in \( \mathbb{R}^3 \) that is not in the range of \( T \).
Square matrices

For a square matrix we can think of the associated matrix transformation

\[ T : \mathbb{R}^n \rightarrow \mathbb{R}^n \]

as doing something to \( \mathbb{R}^n \).

Example. The matrix transformation \( T \) for

\[
\begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix}
\]

What does \( T \) do to \( \mathbb{R}^2 \)?
Square matrices

What does each matrix do to $\mathbb{R}^2$?

\[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
3 & 0 \\
0 & 3
\end{pmatrix}
\]

What is the range in each case?
Poll

What does \[
\begin{pmatrix}
1 & 1 \\
0 & 1
\end{pmatrix}
\] do to this letter F?
Square matrices

What does each matrix do to $\mathbb{R}^2$?

*Hint: if you can’t see it all at once, see what happens to the $x$- and $y$-axes.*

\[
\begin{pmatrix}
1 & 1 \\
0 & 1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & -1 \\
1 & 1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta \\
\end{pmatrix}
\]
Examples in $\mathbb{R}^3$

What does each matrix do to $\mathbb{R}^3$?

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\]
Section 4.1 Summary

- If $A$ is an $m \times n$ matrix, then the associated matrix transformation $T$ is given by $T(v) = Av$. This is a function with domain $\mathbb{R}^n$ and codomain $\mathbb{R}^m$ and range $\text{Col}(A)$.

- If $A$ is $n \times n$ then $T$ does something to $\mathbb{R}^n$; basic examples: reflection, projection, scaling, shear, rotation.