Announcements: November 12

- Midterm 3 on §4.5-6.5 Friday in recitation
- WeBWorK 6.3, 6.4, 6.5 due Wednesday
- My office hours Wed 2-3 and Friday 9:30-10:30 in Skiles 234
- TA office hours
  - Arjun Wed 3-4 Skiles 230
  - Talha Tue/Thu 11-12 Clough 250
  - Athreya Tue 3-4 Skiles 230
  - Olivia Thu 3-4 Skiles 230
  - James Tue 11-12 Skiles 230
  - Jesse Wed 9:30-10:30 Skiles 230
  - Vajraang Thu 11-12 Skiles 230
  - Hamed Thu 11:15-12, 1:45-2:45, 3-4:15 Clough 280
- Math Lab Monday-Thursday 11:15-5:15 Clough 280 Schedule
- Review sessions
  - Talha Wed 6-7:30, Skiles 368
  - Talha Thu 6-7:30, Skiles 269
- PLUS Sessions
  - Tue/Thu 6-7 Westside Activity Room
  - Mon/Wed 6-7 Westside Activity Room

Section 6.6

Stochastic Matrices (and Google!)

Outline of Section 6.6

- Stochastic matrices and applications
- The steady state of a stochastic matrix
- Important web pages

Stochastic matrices

A stochastic matrix is a non-negative square matrix where all of the columns add up to 1.

Examples:

$$\begin{pmatrix} 1/4 & 3/5 \\ 3/4 & 2/5 \end{pmatrix} \quad \begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix} \quad \begin{pmatrix} 1/2 & 1 & 1/2 \\ 1/2 & 0 & 1/4 \\ 0 & 0 & 1/4 \end{pmatrix}$$

Application: Rental Cars

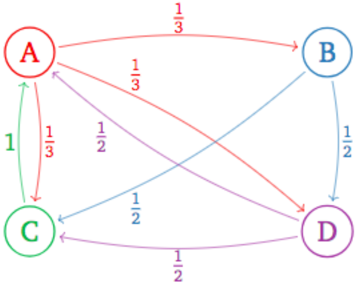
Say your car rental company has 3 locations. Make a matrix whose ij entry is the fraction of cars at location i that end up at location j. For example,

$$\begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix}$$

Note the columns sum to 1. Why?

Application: Web pages

Make a matrix whose ij entry is the fraction of (randomly surfing) web surfers at page i that end up at page j. If page i has N links then the ij-entry is either 0 or $1/N$.



$$\begin{pmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{pmatrix}$$

Properties of stochastic matrices

Let A be a stochastic matrix.

Fact. One of the eigenvalues of A is 1 and all other eigenvalues have absolute value at most 1.

Now suppose A is a positive stochastic matrix.

Fact. The 1-eigenspace of A is 1-dimensional; it has a positive eigenvector.

The unique such eigenvector with entries adding to 1 is called the steady state vector.

Fact. Under iteration, all nonzero vectors approach the steady state vector.

Demo

The last fact tells us how to distribute rental cars, and also tells us the importance of web pages!

Application: Rental Cars

The rental car matrix is:

$$\begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix}$$

Its steady state vector is:

$$\frac{1}{18}\begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix} \approx \begin{pmatrix} .39 \\ .33 \\ .28 \end{pmatrix}$$

Application: Web pages

The web page matrix is:

$$\begin{pmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{pmatrix}$$

Its steady state vector is approximately

$$\begin{pmatrix} .39 \\ .13 \\ .29 \\ .19 \end{pmatrix}$$

and so the first web page is the most important.

Summary of Section 6.6

- A stochastic matrix is a non-negative square matrix where all of the columns add up to 1.
- Every stochastic matrix has 1 as an eigenvalue, and all other eigenvalues have absolute value at most 1.
- A positive stochastic matrix has 1-dimensional eigenspace and has a positive eigenvector.
- For a positive stochastic matrix, a positive 1-eigenvector with entries adding to 1 is called a steady state vector.
- For a positive stochastic matrix, all nonzero vectors approach the steady state vector under iteration.
- Steady state vectors tell us the importance of web pages (for example).