Announcements: November 12

- Final Exam Dec 11 6-8:50p (cumulative!)
- WeBWorK 6.6 (Eigenvalues in Engineering) due Wednesday
- No office hours this week
- TA office hours
  - Arjun Wed 3-4 Skiles 230
  - Talha Tue/Thu 11-12 Clough 250
  - Athreya Tue 3-4 Skiles 230
  - Olivia Thu 3-4 Skiles 230
  - James Tue 11-12 Skiles 230
  - Jesse Wed 9:30-10:30 Skiles 230
  - Vajraang Thu 11-12 Skiles 230
  - Hamed Thu 11:15-12, 1:45-2:45, 3-4:15 Clough 280
- Math Lab Monday-Thursday 11:15-5:15 Clough 280 Schedule
- Review sessions
  - Talha Wed 6-7:30, Skiles 368
  - Talha Thu 6-7:30, Skiles 269
- PLUS Sessions
  - Tue/Thu 6-7 Westside Activity Room
  - Mon/Wed 6-7 Westside Activity Room

Section 7.2

Orthogonal complements

Outline of Section 7.2

- Orthogonal complements
- Computing orthogonal complements

Orthogonal complements

W = subspace of $\mathbb{R}^n$
$W^\perp = \{v \text{ in } \mathbb{R}^n \mid v \perp w \text{ for all } w \text{ in } W\}$

Question. What is the orthogonal complement of a line in $\mathbb{R}^3$?

Demo   Demo

Facts.

1. $W^\perp$ is a subspace of $\mathbb{R}^n$
2. $(W^\perp)^\perp = W$
3. $\dim W + \dim W^\perp = n$
4. If $W = \mathrm{Span}\{w_1, \ldots, w_k\}$ then
   $W^\perp = \{v \text{ in } \mathbb{R}^n \mid v \perp w_i \text{ for all } i\}$
5. The intersection of W and $W^\perp$ is $\{0\}$.

Orthogonal complements

Finding them

Problem. Let $W = \mathrm{Span}\{(1, 1, -1)\}$. Find the equation of the plane $W^\perp$.

Problem. Let $W = \mathrm{Span}\{(1, 1, -1), (-1, 2, 1)\}$. Find a system of equations describing the line $W^\perp$.

Orthogonal complements

Finding them

Problem. Let $W = \mathrm{Span}\{(1,1,-1), (-1,2,1)\}$. Find a system of equations describing the line $W^\perp$.

Theorem. $A = m \times n$ matrix

$$(\mathrm{Row} A)^\perp = \mathrm{Nul}\, A$$

Why? $Ax = 0 \Leftrightarrow x$ is orthogonal to each row of A

Orthogonal decomposition

Fact. Say W is a subspace of $\mathbb{R}^n$. Then any vector v in $\mathbb{R}^n$ can be written uniquely as

$$v = v_W + v_{W^\perp}$$

where v_W is in W and $v_{W^\perp}$ is in $W^\perp$.

Why? Say that $w_1 + w_1' = w_2 + w_2'$ where w_1 and w_2 are in W and w_1' and w_2' are in $W^\perp$. Then $w_1 - w_2 = w_2' - w_1'$. But the former is in W and the latter is in $W^\perp$, so they must both be equal to 0.

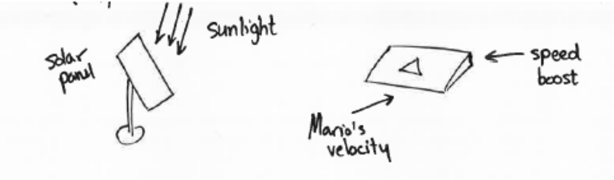
Demo

Demo

Next time: Find v_W and $v_{W^\perp}$.

Orthogonal Projections

Many applications, including:



Summary of Section 7.2

- $W^\perp = \{v \text{ in } \mathbb{R}^n \mid v \perp w \text{ for all } w \text{ in } W\}$
- Facts:
  1. $W^\perp$ is a subspace of $\mathbb{R}^n$
  2. $(W^\perp)^\perp = W$
  3. $\dim W + \dim W^\perp = n$
  4. If $W = \mathrm{Span}\{w_1, \ldots, w_k\}$ then $W^\perp = \{v \text{ in } \mathbb{R}^n \mid v \perp w_i \text{ for all } i\}$
  5. The intersection of W and $W^\perp$ is $\{0\}$.
- $(\mathrm{Row}\, A)^\perp = \mathrm{Nul}\, A$ (this is how you *find* $W^\perp$)
- Every vector v can be written uniquely as $v = v_W + v_{W^\perp}$ with v_W in W and $v_{W^\perp}$ in $W^\perp$