Announcements: November 25

- CIOS: Please do by Monday
- Final Exam Dec 11 6-8:50p (cumulative!)
- Final Exam locations:
  - Section G: Howey L3
  - Section H: Howey L4
- WeBWorK 7.3 & 7.5 not due or graded (for practice only)
- My office hours Wed 2-3 in Skiles 234
- CAS Study Session 3-5 Wed Dec 5 Clough 144
- TA office hours
  - Arjun Wed 3-4 Skiles 230
  - Talha Tue/Thu 11-12 Clough 250
  - Athreya Tue 3-4 Skiles 230
  - Olivia Thu 3-4 Skiles 230
  - James Tue 11-12 Skiles 230
  - Jesse Wed 9:30-10:30 Skiles 230
  - Vajraang Thu 11-12 Skiles 230
  - Hamed Thu 11:15-12, 1:45-2:45, 3-4:15 Clough 280
- Review sessions tba
Review for Final Exam
Summary of Section 6.6

- A stochastic matrix is a non-negative square matrix where all of the columns add up to 1.
- Every stochastic matrix has 1 as an eigenvalue, and all other eigenvalues have absolute value at most 1.
- A positive stochastic matrix has 1-dimensional eigenspace and has a positive eigenvector.
- For a positive stochastic matrix, a positive 1-eigenvector with entries adding to 1 is called a steady state vector.
- For a positive stochastic matrix, all nonzero vectors approach the steady state vector under iteration.
- Steady state vectors tell us the importance of web pages (for example).

Suppose the internet has 3 pages. Page 1 links to 2, page 2 links to 3, and page 3 links to 1 and 2. Rank the web pages from most to least important.
Summary of Section 7.1

- \( u \cdot v = \sum u_i v_i \)
- \( u \cdot u = \|u\|^2 \) (length of \( u \) squared)
- The unit vector in the direction of \( v \) is \( v/\|v\| \).
- The distance from \( u \) to \( v \) is \( \|u - v\| \)
- \( u \cdot v = 0 \Leftrightarrow u \perp v \)

Suppose that \( u \) and \( v \) are vectors in \( \mathbb{R}^n \) and \( u \) is perpendicular to \( v \). Is it possible that \( u + v \) is perpendicular to \( v \)?
Summary of Section 7.2

- \( W^\perp = \{ v \in \mathbb{R}^n \mid v \perp w \text{ for all } w \text{ in } W \} \)
- Facts:
  1. \( W^\perp \) is a subspace of \( \mathbb{R}^n \)
  2. \((W^\perp)^\perp = W\)
  3. \( \dim W + \dim W^\perp = n \)
  4. If \( W = \text{Span}\{w_1, \ldots, w_k\} \) then \( W^\perp = \{ v \in \mathbb{R}^n \mid v \perp w_i \text{ for all } i \} \)
  5. The intersection of \( W \) and \( W^\perp \) is \{0\}.
- \((\text{Row } A)^\perp = \text{Nul } A\)  (this is how you find \( W^\perp \))
- Every vector \( v \) can be written uniquely as \( v = v_W + v_{W^\perp} \) with \( v_W \) in \( W \) and \( v_{W^\perp} \) in \( W^\perp \)

Find the equation of the plane \( W \) in \( \mathbb{R}^3 \) perpendicular to the vector \((-1, 2, 1)\). Find a basis for \( W \). Find a basis for \( W^\perp \).
Summary of Section 7.3

- The **orthogonal projection** of $v$ onto $W$ is $v_W$
- $v_W$ is the closest point in $W$ to $v$.
- The distance from $v$ to $W$ is $\|v_W\|$.
- **Theorem.** Let $W = \text{Col}(A)$. For any $v$, the equation $A^T Ax = A^T v$ is consistent and $v_W$ is equal to $Ax$ where $x$ is any solution.
- **Special case.** If $L = \text{Span}\{u\}$ then $v_L = \frac{u \cdot v}{u \cdot u} u$
- When the columns of $A$ are independent, the standard matrix for orthogonal projection to $\text{Col}(A)$ is $A(A^T A)^{-1} A^T$
- Let $W$ be a subspace of $\mathbb{R}^n$ and let $T: \mathbb{R}^n \to \mathbb{R}^n$ be $T(v) = v_W$. Then
  ▶ $T$ is a linear transformation, etc.
- If $P$ is the standard matrix then
  ▶ The 1–eigenspace of $P$ is $W$ (unless $W = 0$), etc.

Is it true that every diagonalizable matrix with eigenvalues 0 and 1 (and no others) is the standard matrix for orthogonal projection onto a subspace?

Compute the distance from the vector $e_1$ to the plane in $\mathbb{R}^3$ spanned by $(1, 0, 1)$ and $(0, 1, -1)$.
Summary of Section 7.5

- A least squares solution to $Ax = b$ is an $\hat{x}$ in $\mathbb{R}^n$ so that $A\hat{x}$ is as close as possible to $b$.
- The error is $\|A\hat{x} - b\|$.
- The least squares solutions to $Ax = b$ are the solutions to $(A^TA)x = (A^Tb)$.
- To find a best fit line/parabola/etc. write the general form of the line/parabola/etc. with unknown coefficients and plug in the given points to get a system of linear equations in the unknown coefficients.

Find the best fit line to the data points $(1, 0)$, $(2, 1)$, and $(3, 3)$. 
Good luck!