Announcements: November 25

- CIOS: Please do by Monday
- Final Exam Dec 11 6-8:50p (cumulative!)
- Final Exam locations:
  - Section G: Howey L3
  - Section H: Howey L4
- WeBWorK 7.3 & 7.5 not due or graded (for practice only)
- My office hours Wed 2-3 in Skiles 234
- CAS Study Session 3-5 Wed Dec 5 Clough 144
- TA office hours
  - Arjun Wed 3-4 Skiles 230
  - Talha Tue/Thu 11-12 Clough 250
  - Athreya Tue 3-4 Skiles 230
  - Olivia Thu 3-4 Skiles 230
  - James Tue 11-12 Skiles 230
  - Jesse Wed 9:30-10:30 Skiles 230
  - Vajraang Thu 11-12 Skiles 230
  - Hamed Thu 11:15-12, 1:45-2:45, 3-4:15 Clough 280
- Review sessions tba

Review for Final Exam

Summary of Section 6.6

- A stochastic matrix is a non-negative square matrix where all of the columns add up to 1.
- Every stochastic matrix has 1 as an eigenvalue, and all other eigenvalues have absolute value at most 1.
- A positive stochastic matrix has 1-dimensional eigenspace and has a positive eigenvector.
- For a positive stochastic matrix, a positive 1-eigenvector with entries adding to 1 is called a steady state vector.
- For a positive stochastic matrix, all nonzero vectors approach the steady state vector under iteration.
- Steady state vectors tell us the importance of web pages (for example).

Suppose the internet has 3 pages. Page 1 links to 2, page 2 links to 3, and page 3 links to 1 and 2. Rank the web pages from most to least important.

Summary of Section 7.1

- $u \cdot v = \sum u_i v_i$
- $u \cdot u = \|u\|^2$ (length of u squared)
- The unit vector in the direction of v is $v/\|v\|$.
- The distance from u to v is $\|u - v\|$
- $u \cdot v = 0 \Leftrightarrow u \perp v$

Suppose that u and v are vectors in $\mathbb{R}^n$ and u is perpendicular to v. Is it possible that $u + v$ is perpendicular to v?

Summary of Section 7.2

- $W^\perp = \{v \text{ in } \mathbb{R}^n \mid v \perp w \text{ for all } w \text{ in } W\}$
- Facts:
  1. $W^\perp$ is a subspace of $\mathbb{R}^n$
  2. $(W^\perp)^\perp = W$
  3. $\dim W + \dim W^\perp = n$
  4. If $W = \mathrm{Span}\{w_1, \ldots, w_k\}$ then $W^\perp = \{v \text{ in } \mathbb{R}^n \mid v \perp w_i \text{ for all } i\}$
  5. The intersection of W and $W^\perp$ is $\{0\}$.
- $(\mathrm{Row}\, A)^\perp = \mathrm{Nul}\, A$ (this is how you *find* $W^\perp$)
- Every vector v can be written uniquely as $v = v_W + v_{W^\perp}$ with v_W in W and $v_{W^\perp}$ in $W^\perp$

Find the equation of the plane W in $\mathbb{R}^3$ perpendicular to the vector $(-1, 2, 1)$. Find a basis for W. Find a basis for $W^\perp$.

Summary of Section 7.3

- The orthogonal projection of v onto W is v_W
- v_W is the closest point in W to v.
- The distance from v to W is $\|v_{W^\perp}\|$.
- Theorem. Let $W = \mathrm{Col}(A)$. For any v, the equation $A^T Ax = A^T v$ is consistent and v_W is equal to Ax where x is any solution.
- Special case. If $L = \mathrm{Span}\{u\}$ then $v_L = \frac{u \cdot v}{u \cdot u} u$
- When the columns of A are independent, the standard matrix for orthogonal projection to $\mathrm{Col}(A)$ is $A(A^T A)^{-1} A^T$
- Let W be a subspace of $\mathbb{R}^n$ and let $T : \mathbb{R}^n \to \mathbb{R}^n$ be $T(v) = v_W$. Then
  - T is a linear transformation, etc.
- If P is the standard matrix then
  - The 1–eigenspace of P is W (unless $W = 0$), etc.

Is is true that every diagonalizable matrix with eigenvalues 0 and 1 (and no others) is the standard matrix for orthogonal projection onto a subspace?

Compute the distance from the vector e_1 to the plane in $\mathbb{R}^3$ spanned by $(1, 0, 1)$ and $(0, 1, -1)$.

Summary of Section 7.5

- A least squares solution to $Ax = b$ is an $\widehat{x}$ in $\mathbb{R}^n$ so that $A\widehat{x}$ is as close as possible to b.
- The error is $\|A\widehat{x} - b\|$.
- The least squares solutions to $Ax = b$ are the solutions to $(A^T A)x = (A^T b)$.
- To find a best fit line/parabola/etc. write the general form of the line/parabola/etc. with unknown coefficients and plug in the given points to get a system of linear equations in the unknown coefficients.

Find the best fit line to the data points $(1, 0)$, $(2, 1)$, and $(3, 3)$.

Good luck!