Math 1553 Worksheet: 4.5, 5.1-5.3

1. Answer true if the statement is *always* true. Otherwise, answer false.

   a) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

   b) Suppose A is an $n \times n$ matrix and every vector in $\mathbf{R}^n$ can be written as a linear combination of the columns of A. Then A must be invertible.

   c) Suppose A and B are invertible $n \times n$ matrices. Then $A + B$ is invertible and
   $$(A+B)^{-1} = A^{-1} + B^{-1}.$$

2. Find the volume of the parallelepiped naturally formed by $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$.

3. Let $A = \begin{pmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{pmatrix}$.

    a) Compute $\det(A)$ using row reduction.

    b) Compute $\det(A^{-1})$ without doing any more work.

    c) Compute $\det((A^T)^5)$ without doing any more work.

4. Play matrix tic-tac-toe!

    Instead of X against O, we have 1 against 0. The 1-player wins if the final matrix has nonzero determinant, while the 0-player wins if the determinant is zero. You can change who goes first, and you can also modify the size of the matrix.

    Click the link above, or copy and paste the url below:

`http://textbooks.math.gatech.edu/ila/demos/tictactoe/tictactoe.html`

Can you think of a winning strategy for the 0 player who goes first in the 2×2 case? Is there a winning strategy for the 1 player if they go first in the 2×2 case?