Midterm 2

ⓘ This is a preview of the published version of the quiz

Started: Oct 20 at 3:27pm

Quiz Instructions

Once you open this quiz, you will have 75 minutes to submit it. You will have only **one** submission attempt. The quiz must be **submitted** by 7:59 PM (Atlanta time) on Friday, October 16. There are 20 questions after the honor code pledge.

This assessment is open-book and open-note, but not open-internet. You may use my class notes, your instructor's notes, and the ILA textbook at **https://textbooks.math.gatech.edu/ila/ila.pdf (https://textbooks.math.gatech.edu/ila/ila.pdf)** .

However, you may not visit any other websites, use any search engines, or use any calculators or computer aids whatsoever (Interactive Row Reducer, Matlab, Mathematica, Chegg.com, Geogebra, etc.) as you take this assessment.

Question 1 0 pts

Please read and attest to the honor statement below:

I understand that this assessment is open-book and open-note, but not open-internet. I may use my class notes, my instructor's notes, and the ILA textbook at **https://textbooks.math.gatech.edu/ila/ila.pdf (https://textbooks.math.gatech.edu/ila/ila.pdf)** .

However, I will not visit any other websites, use any search engines, or use any calculators or computer aids whatsoever (Matlab, Mathematica, Chegg.com, Geogebra, etc.) as I take this assessment.

This assessment is completely my own work. I will not discuss the answers or any of the contents of this assessment with anyone until the time it is due.

- ◯ I attest to my integrity, and I understand that any suspected violation of this policy may be prosecuted to the fullest extent allowable by Georgia Tech.

Question 2

1 pts

Let V be the set of vectors in $\mathbb{R}^3$ given by

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ in } \mathbb{R}^3 : x = 2y \text{ and } z \geq 0 \right\}.$$

(a) Does V contain the 0 vector? [Select]

(b) Is V closed under addition? In other words, if u and v are in V does it follow that $u + v$ is in V? [Select]

(c) Is V closed under scalar multiplication? In other words if v is in V and c is a real number does it follow that cv is in V? [Select]

(d) Is V a subspace of $\mathbb{R}^3$? [Select]

Question 3

1 pts

Consider the matrix

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & -1 & 1 \end{pmatrix}.$$

Complete the following statements concerning the matrix transformation $T(v) = Av$.

(a) The domain of T is $\mathbb{R}^a$, where a= [Select]

(b) The codomain of T is $\mathbb{R}^b$, where b= [Select]

(c) The range of T is [Select]

Question 4

1 pts

Consider the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{pmatrix}$$

and consider the matrix transformation $T(v) = Av$. Which of the following vectors is in the range of T? Select all that apply.

- ☐ $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$
- ☐ $\begin{pmatrix} 1 & 5 \end{pmatrix}$
- ☐ $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
- ☐ $\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$
- ☐ $\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$

Question 5

1 pts

Find a 3×2 matrix in reduced row echelon form so that the associated matrix transformation is one-to-one but not onto.

Question 6

1 pts

Find the value of h so that the following matrix is not invertible.

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & h & h \end{pmatrix}$$

Question 7

1 pts

Let $S : \mathbb{R}^3 \to \mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformations

$$S\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-y-z \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-y \\ y-z \\ z-x \end{pmatrix}$$

Which of the following statements are true? Select all that apply.

- [ ] T is not onto
- [ ] S is one-to-one
- [ ] $T\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\0\\1\end{pmatrix}$
- [ ] $S\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\0\\0\end{pmatrix}$
- [ ] $T \circ S\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\0\\1\end{pmatrix}$

Question 8

1 pts

Suppose that $T : \mathbb{R}^{10} \to \mathbb{R}^{1}$ is a linear transformation and that its standard matrix is not the zero matrix. Which of the following statements must be true? *Select all that apply.*

- [ ] T is one-to-one
- [ ] T is onto
- [ ] T is invertible
- [ ] T is not one-to-one
- [ ] T is not onto
- [ ] T is not invertible

Question 9

1 pts

Which of the following functions are linear transformations? *Select all that apply.*

- ☐ $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
- ☐ $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sin(x) \\ e^y \end{pmatrix}$
- ☐ $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ y \\ x \end{pmatrix}$
- ☐ $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ y \\ 1 \end{pmatrix}$

Question 10

1 pts

Suppose A is a 8×12 matrix. Determine whether each of the following two statements is true or false.

The dimension of the null space of A must be at least 4.

[Select]

The dimension of the column space of A must be at least 8.

[Select]

Question 11

1 pts

Let A be a 5×4 matrix.

(a) The column space of A is a subspace of $\mathbb{R}^a$ where $a =$

[Select] .

(b) The null space of A is a subspace of $\mathbb{R}^b$ where $b =$

[Select] .

(c) Is it possible for $\mathrm{Col}(A)$ to equal $\mathbb{R}^a$? [Select]

Question 12

1 pts

Consider the following matrix.

$$\begin{pmatrix} 1 & 0 & -3 & 0 & 2 \\ 0 & 1 & 14 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

What is the rank of the matrix?

How many vectors are there in a basis for the null space of the matrix?

Question 13

1 pts

Suppose A is an invertible 3×3 matrix and that its inverse is

$$A^{-1} = \begin{pmatrix} -2 & -1 & 3 \\ 1 & 0 & -3 \\ -1 & 2 & 4 \end{pmatrix}.$$

Solve the matrix equation $\boldsymbol{Ax} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

Question 14

1 pts

Suppose we have three linear transformations

$T : \mathbb{R}^2 \to \mathbb{R}^3$
$U : \mathbb{R}^2 \to \mathbb{R}^4$
$V : \mathbb{R}^3 \to \mathbb{R}^3$

Which of the following compositions make sense? *Select all that apply.*

- ☐ $T \circ U$
- ☐ $T \circ V$
- ☐ $V \circ V$
- ☐ $V \circ T$

Question 15

1 pts

Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the projection onto the x-axis and $U : \mathbb{R}^2 \to \mathbb{R}^2$ is the reflection across the line $y = -x$. What is the standard matrix for $T \circ U$?

Question 16

1 pts

Which of the following linear transformations is the matrix transformation for $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$?

- ◯ Clockwise rotation by π radians
- ◯ Dilation by -1 followed by projection onto the x-axis
- ◯ Reflection across the line $y = -x$
- ◯ Dilation by -1 followed by projection onto the y-axis
- ◯ Reflection about the line $y = x$

Question 17

1 pts

Determine whether each of the following statements is true or false.

There is a 3×2 matrix where the column space and the null space are both lines.

[Select]

There is a 2×2 matrix where the column space and the null space are both lines.

[Select]

There is a 2×3 matrix where the column space and the null space are both planes.

[Select]

There is a 3×3 matrix where the column space and the null space are both planes.

[Select]

Question 18

1 pts

Solve for the matrix X, assuming that all matrices arising in the problem are invertible $n \times n$ matrices.

$AX = BX + C$

- $C(A-B)^{-1}$
- $C(B-A)^{-1}$
- $(A-B)^{-1}C$
- $(B-A)^{-1}C$

Question 19

1 pts

Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation and that

$T\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix}$ and $T\begin{pmatrix}1\\0\\-1\end{pmatrix}=\begin{pmatrix}5\\1\end{pmatrix}$.

Compute $T\begin{pmatrix}1\\0\\0\end{pmatrix}$.

Question 20

1 pts

Determine whether each of the following three statements is true or false.

If V is a 1-dimensional subspace of $\mathbb{R}^3$, then every nonzero vector in V forms a basis for V. [Select]

If vectors v and w in $\mathbb{R}^3$ are not collinear, then they form a basis for a plane in $\mathbb{R}^3$.
[Select]

If v_1, v_2, v_3, and v_4 span $\mathbb{R}^4$, then they form a basis for $\mathbb{R}^4$.
[Select]

Question 21

1 pts

Suppose that A and B are $n \times n$ matrices. Let $T(v) = Av$ and $U(v) = Bv$ be the associated matrix transformations. Determine whether each of the following statements is true or false.

If $AB = I_n$, then $Bx = b$ is consistent for every b in $\mathbb{R}^n$.

[Select]

If the columns of A are linearly independent and the columns of B are linearly dependent then $T \circ U$ is not onto. [Select]

If A and B are both invertible then $T \circ U$ is one-to-one.

[Select]

If $Ax = b$ is consistent for every b in $\mathbb{R}^n$, then $Ax = 0$ has infinitely many solutions.

[Select]

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