Midterm 3

This is a preview of the published version of the quiz

Started: Nov 24 at 2:58pm

Quiz Instructions

Once you open this quiz, you will have 75 minutes to submit it. You will have only **one** submission attempt. The quiz must be **submitted** by 7:59 PM (Atlanta time) on Friday, November 20. There are 20 questions after the honor code pledge.

This assessment is open-book and open-note, but not open-internet. You may use your class notes, your instructor's notes, and the ILA textbook at **https://textbooks.math.gatech.edu/ila/ila.pdf (https://textbooks.math.gatech.edu/ila/ila.pdf)** .

However, you may not visit any other websites, use any search engines, or use any calculators or computer aids whatsoever (Interactive Row Reducer, Matlab, Mathematica, Chegg.com, Geogebra, etc.) as you take this assessment.

Recently, Canvas has had issues displaying math equations. If this happens, you should try reloading (before you start!), switching browsers (again, before you start), and/or turning off ad blockers. If none of these work, you can still figure out what is supposed to be displayed. For instance,

Latex: \left(\begin{array}{rr} 1 & 2 \\ 3 & 4 \end{array}\right) is the 2 x 2 matrix with 1 2 on the top row and 3 4 on the bottom.

If it becomes necessary, make a good faith effort to interpret the code as above. If you feel that these technical issues are hindering your ability to take the exam, you should let your instructor know.

Question 1 — 0 pts

Please read and attest to the honor statement below:

I understand that this assessment is open-book and open-note, but not open-internet. I may use my class notes, my instructor's notes, and the ILA textbook at **https://textbooks.math.gatech.edu/ila/ila.pdf (https://textbooks.math.gatech.edu/ila/ila.pdf)** .

However, I will not visit any other websites, use any search engines, or use any calculators or computer aids whatsoever (Matlab, Mathematica, Chegg.com,

Geogebra, etc.) as I take this assessment.

This assessment is completely my own work. I will not discuss the answers or any of the contents of this assessment with anyone until the time it is due.

- ○ I attest to my integrity, and I understand that any suspected violation of this policy may be prosecuted to the fullest extent allowable by Georgia Tech.

Question 2

1 pts

Find the characteristic polynomial of the matrix

$$\begin{pmatrix} 6 & 9 \\ -4 & -6 \end{pmatrix}$$

[ ] $\lambda^2 +$ [ ] $\lambda +$ [ ]

Question 3

1 pts

Find the steady state vector for the following matrix. *The entries of the answer must either be reduced fractions or in decimal form with at most two decimal places.*

$$\begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}$$

[ ]

[ ]

Question 4

1 pts

Suppose A is a 2×2 matrix whose determinant is $1/2$ and whose cofactor matrix equals

$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$.

Solve $Ax = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Question 5

1 pts

Suppose $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is an eigenvector of A with eigenvalue 0, and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ is an eigenvector of A with eigenvalue 2. Find the matrix A.

Question 6

1 pts

Find the value of k so that all eigenvalues of the matrix $\begin{pmatrix} 3 & k \\ 2 & -1 \end{pmatrix}$ are real, but the matrix is not diagonalizable.

Question 7

1 pts

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

.

Enter below *one* of the eigenvalues for the standard matrix for T.

One eigenvalue is $+ \; i$

Question 8

1 pts

Suppose that the determinant of

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

is 1. Find the determinant of

$\begin{pmatrix} 3c - 2a & 3d - 2b \\ 2a & 2b \end{pmatrix}$.

Question 9

1 pts

What is the area of the triangle in $\mathbb{R}^2$ with vertices $(1,2)$, $(3,6)$, and $(3,5)$?

Question 10

1 pts

Find the value of k so that the following matrix has determinant 8.

$$\begin{pmatrix} 2 & 4 & 4 \\ 0 & 3 & k \\ -1 & k & -1 \end{pmatrix}$$

Question 11

1 pts

Let A be a 2×2 matrix whose trace is 5 and whose determinant is 0.

The smaller eigenvalue of A is _____ .

The larger eigenvalue of A is _____ .

Question 12

1 pts

Consider the matrix $\begin{pmatrix} 0 & -2 \\ 1 & 2 \end{pmatrix}$. Which of the following is an eigenvector for the eigenvalue $1+i$?

- ○ $\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$
- ○ $\begin{pmatrix} -2 \\ 1+i \end{pmatrix}$
- ○ $\begin{pmatrix} -2 \\ 1-i \end{pmatrix}$
- ○ $\begin{pmatrix} 2 \\ 1-i \end{pmatrix}$

Question 13

1 pts

Let $A = \begin{pmatrix} 5 & 7 \\ 6 & 4 \end{pmatrix}$ and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the associated linear transformation. If S is the square with vertices $(0,0)$, $(0,1)$, $(1,0)$, and $(1,1)$, find the area of $T(S)$.

Question 14

1 pts

Which of the following matrices has characteristic polynomial $\lambda^2 - 1$? *Select all that apply.*

- ☐ $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
- ☐

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

- [ ] $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
- [ ] $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Question 15

1 pts

Is there a 4×4 matrix (with real entries) that has 0, $1+i$, and $-1+i$ as eigenvalues? [Select]

Question 16

1 pts

Suppose that A is a 2×2 stochastic matrix with steady state vector

$\frac{1}{32}\begin{pmatrix} 12 \\ 20 \end{pmatrix} = \begin{pmatrix} 12/32 \\ 20/32 \end{pmatrix}$.

To what vector does $A^n \begin{pmatrix} 34 \\ 30 \end{pmatrix}$ approach as n tends to infinity?

Question 17

1 pts

(a) Suppose A is a 2×2 matrix that has only one eigenvalue and its geometric multiplicity 1. Is A diagonalizable? [Select]

(b) Suppose B is a 4×4 matrix with exactly three eigenvalues -1, 0, and 1. Is B diagonalizable? [Select]

(c) Suppose C is a 3×3 upper triangular matrix whose diagonal entries are 2, 3, and -4. Is C diagonalizable? [Select]

Question 18

1 pts

Let A be the 3×3 diagonalizable matrix

$$A = \begin{pmatrix} 0 & 0 & -4 \\ 2 & 2 & 2 \\ -6 & -6 & -2 \end{pmatrix}$$

which we can write as $A = CDC^{-1}$ where

$$C = \begin{pmatrix} -1 & 3 & -1 \\ 0 & -2 & 1 \\ 1 & 3 & 0 \end{pmatrix} \text{ and } D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}.$$

Then

$\lambda_1 =$

$\lambda_2 =$

$\lambda_3 =$

Question 19

1 pts

Consider the matrix $\begin{pmatrix} -1 & 3 & -1 \\ 0 & -2 & 1 \\ 1 & 3 & 0 \end{pmatrix}$. Which of the following is a basis for the eigenspace with eigenvalue -2?

- $\left\{ \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \right\}$

Question 20

1 pts

Suppose that A is a 4×4 matrix with eigenvalues -3, -1, 2, and 4. Determine whether each of the following statements is true or false.

(a) A also has 1 as an eigenvalue. [Select]

(b) A is invertible. [Select]

(c) If v_1 is an eigenvector for -1 and v_2 is an eigenvector for 4, then v_1 and v_2 are linearly independent. [Select]

Question 21

1 pts

Which of the following mathematical statements implies that $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ is an eigenvector for a matrix A? *Select all that apply.*

- ☐ $A\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
- ☐ $A\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$
- ☐ $A\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$
- ☐ $A\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} \sqrt{50} \\ -\sqrt{2} \end{pmatrix}$
- ☐ $A\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

Not saved Submit Quiz