Practice Midterm 3

ⓘ This is a preview of the published version of the quiz

Started: Nov 18 at 2:08pm

Quiz Instructions

Question 1

1 pts

If A is an $n \times n$ matrix, then the determinant of A is the same as the determinant of the reduced row echelon form of A.

- True
- False

Question 2

1 pts

If A is a 3×3 matrix with characteristic polynomial

$\det(A - \lambda I) = (1 - \lambda)(-1 - \lambda)^2$,

then A must be invertible.

- True
- False

Question 3

1 pts

Find the cofactor expansion for the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

corresponding to the top row: $a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$.

[ ] [ ] + [ ] [ ] +
[ ] [ ]

Question 4

1 pts

If A and B are 3×3 matrices that have the same eigenvalues and the same algebraic multiplicity for each eigenvalue, then $A = B$.

- True
- False

Question 5

1 pts

Suppose A is an $n \times n$ matrix and λ is an eigenvalue of A. If v and w are two different eigenvectors of A corresponding to λ, then $v - w$ is an eigenvector of A.

- True
- False

Question 6

1 pts

Say that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that projects onto the x-axis, and that A is the standard matrix for T.

Write the eigenvalues of A, **in increasing order**.

The eigenvalues are ______ and ______ .

Question 7

1 pts

Suppose $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 3.$

Find $\det \begin{pmatrix} -4a+d & -4b+e & -4c+f \\ a & b & c \\ g & h & i \end{pmatrix}.$

Question 8

1 pts

Suppose A is a 3×3 matrix and $\det(A) = 2$. Find $\det(-2A^{-1})$.

Enter an integer as your answer.

Question 9

1 pts

Find the eigenvalues of this matrix: $\begin{pmatrix} -1 & 0 \\ 4 & 5 \end{pmatrix}$

The eigenvalues are [ ] and [ ].

Question 10

1 pts

Consider the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

Which of the following is true?

- The matrix is diagonalizable but not invertible.
- The matrix is invertible but not diagonalizable.
- The matrix is invertible and diagonalizable.
- The matrix is neither invertible nor diagonalizable.

Question 11

1 pts

Let A be the 2×2 matrix that which implements reflection in $\mathbf{R}^2$ across the line $y = -\frac{x}{2}$.

Write one vector in the (-1)-eigenspace of A.

- $\begin{pmatrix} -1/2 \\ 1 \end{pmatrix}$

- ○ $\begin{pmatrix} 1 \\ -1/2 \end{pmatrix}$
- ○ $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
- ○ $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Question 12

1 pts

Suppose A is an $n \times n$ matrix and $\det(A) = 0$.

Which of the following statements must be true? Select all that apply.

- ☐ $\lambda = 0$ is an eigenvalue of A.
- ☐ $\dim(\mathrm{Nul}(A)) \geq 1$
- ☐ The equation $Ax = 0$ has only the trivial solution $x = 0$.

Question 13

1 pts

Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation whose standard matrix is

$$A = \begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix}.$$

Find the area of $T(S)$, where S is a square with area 2.

Enter an integer as your answer.

Question 14

1 pts

Consider the matrix $A = \begin{pmatrix} c & 0 & 1 \\ c & c & 4 \\ 2 & 0 & 1 \end{pmatrix}$.

Find all values of c (if there are any) so that $\det(A) = 0$.

- c = 0 and c = 2
- c = 0 only
- c= -1 only
- c = 2 only
- c = 1 only
- There is no value of c that makes the determinant 0.
- c = 1 and c = -1

Question 15

1 pts

Let $A = \begin{pmatrix} 3 & 0 & -2 \\ 2 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$.

Find all eigenvalues of A.

- 1 only
- 3 only
- -1 only

- -1 and -3
- 2 and -2
- 1 only
- 1 and 3

Question 16

1 pts

Suppose A is a 2×2 matrix satisfying $\mathbf{Tr}(A) = 6$ and $\det(A) = 9$.

Which of the following is true?

- A is diagonalizable but A cannot be invertible.
- It is not possible to determine whether A is invertible or diagonalizable from the information given.
- A is invertible, but A is not necessarily diagonalizable.
- A must be invertible and diagonalizable.
- A must be diagonalizable, but A is not necessarily invertible.
- A is invertible but A cannot be diagonalizable.
- A is neither invertible nor diagonalizable.

Question 17

1 pts

Find a basis for the 2-eigenspace of $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.

-

$$\left\{\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}.$$

- $\left\{\begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$
- $\left\{\begin{pmatrix}2\\1\\0\end{pmatrix},\begin{pmatrix}0\\2\\1\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$
- $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\right\}.$
- none of these
- $\left\{\begin{pmatrix}2\\1\\0\end{pmatrix},\begin{pmatrix}0\\2\\1\end{pmatrix}\right\}$

Question 18

1 pts

Let $A=\begin{pmatrix}1&1\\0&-3\end{pmatrix}\begin{pmatrix}1&0\\0&3/2\end{pmatrix}\begin{pmatrix}1&1\\0&-3\end{pmatrix}^{-1}$. Find the eigenvalues of A.

- 3 and 1/2
- 3 only
- 1 only
- 3/2 only
- 1 and 3/2
- 1 and -1
- 1/2 and 3/2

Question 19

1 pts

Suppose $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -1/3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}^{-1}$.

Which of the following statements is true? Select only one answer.

- ○ Every nonzero vector in $\mathbf{R}^2$ is an eigenvector of A.
- ○ If $x = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, then $A^n x$ approaches the zero vector as n becomes very large.
- ○ Repeated multiplication by A pushes vectors towards $\mathrm{Span}\left\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$.

Question 20

1 pts

Courage Soda and Dexter Soda compete for a market of 210 customers who drink soda each day.

Today, Courage has 80 customers and Dexter has 130 customers.

Each day:

70% of Courage Soda's customers keep drinking Courage Soda, while 30% switch to Dexter Soda.

40% of Dexter Soda's customers keep drinking Dexter Soda, while 60% switch to Courage Soda.

Find a stochastic matrix A and a vector x so that Ax will give the number of customers for Courage Soda and Dexter Soda (in that order) tomorrow.

- ○ $A = \begin{pmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{pmatrix}, \quad x = \begin{pmatrix} 210 \\ 210 \end{pmatrix}.$

- $A = \begin{pmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{pmatrix}, \quad x = \begin{pmatrix} 80 \\ 130 \end{pmatrix}.$
- $A = \begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{pmatrix}, \quad x = \begin{pmatrix} 80 \\ 130 \end{pmatrix}.$
- $A = \begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{pmatrix}, \quad x = \begin{pmatrix} 130 \\ 80 \end{pmatrix}.$
- $A = \begin{pmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{pmatrix}, \quad x = \begin{pmatrix} 130 \\ 80 \end{pmatrix}.$
- none of these
- $A = \begin{pmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{pmatrix}, \quad x = \begin{pmatrix} 210 \\ 210 \end{pmatrix}.$
- $A = \begin{pmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{pmatrix}, \quad x = \begin{pmatrix} 130 \\ 80 \end{pmatrix}.$
- $A = \begin{pmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{pmatrix}, \quad x = \begin{pmatrix} 80 \\ 130 \end{pmatrix}.$

Question 21

1 pts

Suppose $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}.$

Find the number c so that the vector $\begin{pmatrix} c \\ i\sqrt{2} \end{pmatrix}$ the is an eigenvector corresponding to the eigenvalue $\lambda = 1 - i\sqrt{2}$.

Question 22

1 pts

Let $A = \begin{pmatrix} \cos(17^\circ) & -\sin(17^\circ) \\ \sin(17^\circ) & \cos(17^\circ) \end{pmatrix}$.

Which of the following is true about the eigenvalues of A?

- A has one real eigenvalue and one complex eigenvalue
- A has two distinct complex eigenvalues
- A has no real eigenvalues and no complex eigenvalues.
- A has one real eigenvalue with algebraic multiplicity 2
- A has two distinct real eigenvalues

Question 23

1 pts

Let A be the 2×2 matrix whose 2-eigenspace is the line $x_2 = 3x_1$ and whose null space is the line $x_2 = -x_1$.

Write A as the product CDC^{-1} where D is a diagonal 3×3 matrix.

- $A = \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix}^{-1}$
- $A = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix}^{-1}$
- $A = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix}^{-1}$
- $A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}^{-1}$
-

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}^{-1}$$

- ○ $$A = \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix}^{-1}$$
- ○ $$A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}^{-1}$$

Question 24

1 pts

Consider an internet with three pages 1, 2, and 3.

Page 1 links to pages 2 and 3.

Page 2 links only to page 3.

Page 3 links to pages 1 and 2.

What is the importance matrix for this internet?

- ○ $$A = \begin{pmatrix} 1 & 0 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1 & 1 \end{pmatrix}.$$
- ○ $$A = \begin{pmatrix} 0 & 0 & 1/3 \\ 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 0 \end{pmatrix}.$$
- ○ $$A = \begin{pmatrix} 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 2/3 & 1/3 \end{pmatrix}.$$
- ○ $$A = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}.$$
- ○

$$A = \begin{pmatrix} 0 & 0 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1 & 0 \end{pmatrix}.$$

Question 25

1 pts

The city of Townsville has 600 inhabitants who eat each day at one of the three restaurants X, Y, and Z. The eating habits of the population are modeled by a positive stochastic matrix whose 1-eigenspace is spanned by $\begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix}$.

Today, X has 100 daily patrons, Y has 0 daily patrons, and Z has 500 daily patrons. In the long run, roughly how many daily patrons will restaurant Y have?

Enter an integer as your answer.

Not saved Submit Quiz