Q1: False. Row multiplication can charg determinant.

Q2: True: since $\det(A-\lambda I) = (1-\lambda)(-1-\lambda)^2$ A does not have zero as an eigenvalue.

Q3: $1 \cdot \det\begin{pmatrix} 5 & 6 \\ 8 & 9 \end{pmatrix} + 2 \cdot (-1) \det\begin{pmatrix} 4 & 6 \\ 7 & 9 \end{pmatrix} + 3 \cdot \det\begin{pmatrix} 4 & 5 \\ 7 & 8 \end{pmatrix}$

$= 1 \cdot (-3) + 2 \cdot 6 + 3 \cdot (-3)$.

Q4: False: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ have the same cheracterstrc polynomal,

Q5: True: since v and w are different $v - w \neq 0$ and $A(v-w) = Av - Aw = \lambda(v-w)$.

Q6: 0 and 1: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is the matrix for T

Q7: -3: $\det\begin{pmatrix} -4a+d & -4b+e & -4c+f \\ a & b & c \\ g & h & i \end{pmatrix} = \det\begin{pmatrix} d & e & f \\ a & b & c \\ g & h & i \end{pmatrix}$

$= -\det\begin{pmatrix} d & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$.

Q8: -4: $\det(-2A^{-1}) = (-2)^3 \det(A^{-1}) = (-2)^3 \det(A)^{-1}$

$= -8 \cdot 2^{-1} = -4$.

Q9: -1 and 5: $\begin{pmatrix} -1 & 0 \\ 4 & 5 \end{pmatrix}$ is lower triangular so the diagonal entries are eigenvalues.

Monday, November 16, 2020 1:21 PM

10) $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

$\det(A) = 1 \det\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} = 1 \cdot 1 \cdot 3 = 3$

$\rightarrow$ A is invertible

finding eigenvalues:

$\lambda = 1$ (mult 2), 3

$(A - 1I)\vec{v} = \vec{0}$

$\begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \vec{v} = \vec{0}$

$x_1 = x_1$

$x_2 = x_2$

$2x_3 = 0$

$\vec{x} = x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$(A - 3I)\vec{v} = \vec{0}$

$\begin{pmatrix} -2 & 0 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \vec{v} = \vec{0}$

$-2x_1 + 2x_3 = 0 \rightarrow x_1 = x_3$

$-2x_2 = 0 \rightarrow x_2 = 0$

$x_3 = x_3$

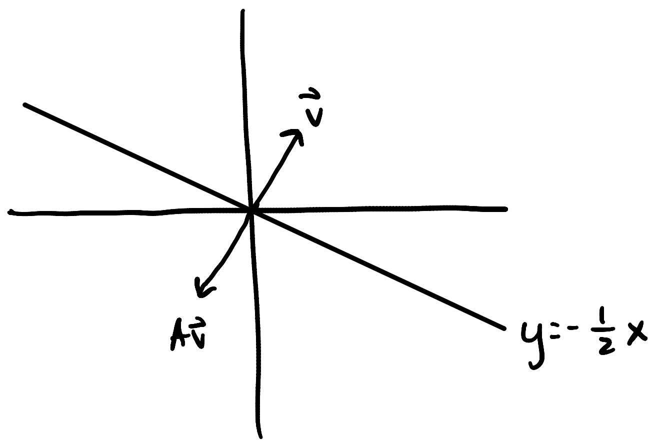
$\vec{x} = x_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

there are 3 distinct eigenvectors

$\rightarrow$ A is diagonizable

11)



-1 eigenspace is perpendicular to line $\rightarrow$ $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

12) A is $n \times n$, $\det(A) = 0$

$\det(A) = 0$ $\rightarrow$ matrix is not invertible
$\rightarrow$ columns are not linearly independent

a) The equation $Ax = 0$ has only the trivial solution $x = 0$.

$\rightarrow$ False, only holds if columns are linearly independent

b) $\lambda = 0$ is an eigenvalue of A.

$\rightarrow$ True, $\det(A - (0)I) = \det(A) = 0$ ✓

c) $\dim(\mathrm{Nul}(A)) \geq 1$

$\rightarrow$ True, by consequence of a)

13) $T: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad$ area$(S) = 2$

$$A = \begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix}$$

$$\begin{aligned} \text{area}(T(S)) &= |\det(A)| \cdot \text{area}(S) \\ &= |2 \cdot 2 - 3 \cdot 5| \cdot (2) \\ &= |4 - 15| \cdot 2 \\ &= 11 \cdot 2 \\ &= 22 \end{aligned}$$

14) $A = \begin{pmatrix} c & 0 & 1 \\ c & c & 4 \\ 2 & 0 & 1 \end{pmatrix}$

want $\det(A) = 0$

$\det(A) = c \det \begin{pmatrix} c & 1 \\ 2 & 1 \end{pmatrix}$ cofactor expansion on 2nd column

$= c(c - 2)$

$c(c-2) = 0$

$c = 0, 2$

15) $A = \begin{pmatrix} 3 & 0 & -2 \\ 2 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

find eigenvalues:

$\det(A - \lambda I) = 0$

$$\det \begin{pmatrix} 3-\lambda & 0 & -2 \\ 2 & 1-\lambda & -2 \\ 0 & 0 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda) \det \begin{pmatrix} 3-\lambda & 0 \\ 2 & 1-\lambda \end{pmatrix} = 0$$ cofactor expansion on 3^{rd} row

$$(1-\lambda)\left[(3-\lambda)(1-\lambda)\right] = 0$$

$\underline{\lambda = 1, 3}$

16) A is 2×2

$Tr(A) = 6$

$\det(A) = 9$

$\det(A) = 9 \neq 0 \rightarrow$ <u>A is invertible</u>

A is <u>not necessarily diagonizable</u>

consider: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ vs $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$

17) $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

$(A - 2I)\vec{v} = \vec{0}$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \vec{v} = 0$$

$x_1 = x_1$

$x_2 = 0$ $\qquad \vec{x} = x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$x_3 = 0$

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

18) $A = \begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3/2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix}^{-1}$

this is in diagonalized form (CDC^{-1})

where diagonal of D = eigenvalues

$\lambda = 1, \ 3/2$

19) $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -1/3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}^{-1}$

a) Repeated multiplication by A pushes vectors towards span$\left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$

consider any vector x. x can be written as a linear combination of the eigenvectors $\left(v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right)$

$$X = a_1 v_1 + a_2 v_2 \quad ①$$

repeated multiplication of A $= \lim_{n \to \infty} A^n x$

$$\lim_{n\to\infty} A^n x = \lim_{n\to\infty} A^n (a_1 v_1 + a_2 v_2) = \lim_{n\to\infty} a_1 A^n v_1 + a_2 A^n v_2$$

$$= \lim_{n\to\infty} a_1 \lambda_1^n v_1 + a_2 \lambda_2^n v_2 = \lim_{n\to\infty} a_1 \left(-\tfrac{1}{3}\right)^n \begin{pmatrix} 1 \\ -1 \end{pmatrix} + a_2 (1)^n \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= 0 + a_2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = a_2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad ②$$

False

b) if $x = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, then $A^n x \to \vec{0}$ as $n \to \infty$

using ① above, $x = \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$a_2 = 0$

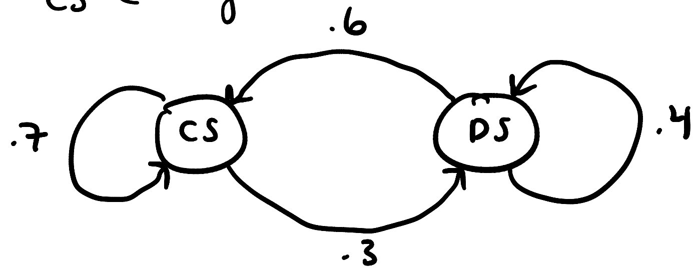
using ② above, $A^n x = 0 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \vec{0}$

True

c) Every nonzero vector in $\mathbb{R}^2$ is an eigenvector of A

False, only span$\left\{\begin{pmatrix}1\\-1\end{pmatrix}\right\}$ or span$\left\{\begin{pmatrix}2\\3\end{pmatrix}\right\}$

20) CS - Courage Soda DS - Dexter Soda



$$A = \begin{array}{c} \\ \text{to} \\ CS \\ DS \end{array} \begin{array}{c} \text{from} \\ \begin{array}{cc} CS & DS \end{array} \\ \begin{pmatrix} .7 & .6 \\ .3 & .4 \end{pmatrix} \end{array}$$

$$x = \begin{array}{c} CS \\ DS \end{array} \begin{pmatrix} 80 \\ 130 \end{pmatrix}$$

21) $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$

$\lambda = 1 - i\sqrt{2}$

$(A - \lambda I)\vec{v} = \vec{0}$

$$\begin{pmatrix} 1-(1-i\sqrt{2}) & -1 \\ 2 & 1-(1-i\sqrt{2}) \end{pmatrix} \vec{v} = \vec{0}$$

$$\begin{pmatrix} i\sqrt{2} & -1 \\ 2 & i\sqrt{2} \end{pmatrix} \vec{v} = 0$$

remember $\begin{pmatrix} z & w \\ * & * \end{pmatrix} \Rightarrow \begin{pmatrix} -w \\ z \end{pmatrix}$ is an eigenvector

so $\vec{v} = \begin{pmatrix} 1 \\ i\sqrt{2} \end{pmatrix}$

$\underline{c = 1}$

22) $A = \begin{pmatrix} \cos 17° & -\sin 17° \\ \sin 17° & \cos 17° \end{pmatrix}$

method i) $\det(A - \lambda I) = 0$

$$\det \begin{pmatrix} \cos 17° - \lambda & -\sin 17° \\ \sin 17° & \cos 17° - \lambda \end{pmatrix} = 0$$

$$(\cos 17° - \lambda)^2 + \sin^2 17° = 0$$

$$\lambda^2 - 2\cos 17° \lambda + \cos^2 17° + \sin^2 17° = 0$$

$$\lambda^2 - 2\cos 17° \lambda + 1 = 0$$

$$\frac{2\cos 17° \pm \sqrt{4\cos^2 17° - 4(1)(1)}}{2} = \lambda$$

notice $0 < \cos^2 17° < 1$

so $4\cos^2 17° - 4 < 0$

$\therefore$ <u>two distinct complex eigenvalues</u>

method ii) notice A is a counterclockwise transformation by $17°$

$\rightarrow$ Ax cannot be colinear with any vector x (ie $Ax \neq \lambda x$)

$\therefore$ there are no real eigenvalues and there must be two distinct complex eigenvalues

23) A is 2×2

2-eigenspace is $x_2 = 3x_1$

null-space is $x_2 = -x_1$

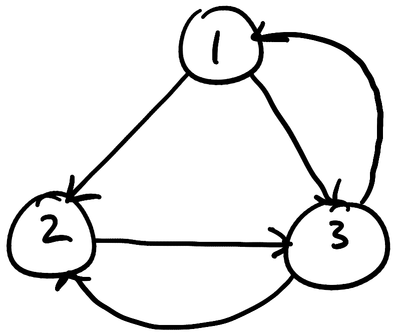
line $x_2 = 3x_1 \rightarrow \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is eigenvector for $\lambda = 2$

Existence of null-space implies $\lambda = 0$ is an eigenvalue.

line $x_2 = -x_1 \rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is eigenvector for $\lambda = 0$

$$A = \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix}^{-1}$$

24)



1 – 2 links → passes $\frac{1}{2}$ importance to 2, 3

2 – 1 link → passes 1 importance to 3

3 – 2 links → passes $\frac{1}{2}$ importance to 1, 2

$$A = \begin{array}{c c} & \text{from} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{to} & \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1 & 0 \end{pmatrix} \end{array}$$

25) for stochastic matrix, long run will approach span{1-eigenspace}

→ long run approaches $x\begin{pmatrix}6\\3\\9\end{pmatrix} = \begin{pmatrix}6x\\3x\\9x\end{pmatrix}$

and we want $6x + 3x + 9x = 600$

$x = 33.3$

long run $= \begin{pmatrix}200\\100\\300\end{pmatrix}$

$Y = \underline{100}$ patrons

notice initial daily patrons does not matter