Practice Final Exam

This is a preview of the published version of the quiz

Started: Nov 30 at 7:55pm

Quiz Instructions

Question 1

1 pts

If $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors in $\mathbf{R}^4$ then $\{v_1+v_3, v_2+v_3, v_3\}$

is linearly independent.

$c_1(v_1+v_3)+c_2(v_2+v_3)+c_3v_3=0$

$\Leftrightarrow c_1v_1+c_2v_2+(c_1+c_2+c_3)v_3=0$

v_1, v_2, v_3 indep $\Rightarrow c_1=c_2=(c_1+c_2+c_3)=0$

$\Rightarrow c_1=c_2=c_3=0$

- True ✓
- False

Question 2

1 pts

Suppose A, B, and C are $n \times n$ matrices.

If A is invertible and $AB=AC$, then $B=C$.

$A^{-1}AB=A^{-1}AC$

- True ✓
- False

Question 3

1 pts

If A is a 6×4 matrix, then the equation $Ax = 0$ must have a non-trivial solution.

6 equations 4 variables

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \\ & & & \\ & & & \end{bmatrix}$$

no free

- True
- False ✓

Question 4

1 pts

If an $n \times n$ matrix A has two eigenvectors u and v corresponding to the same eigenvalue λ, then u must be a scalar multiple of v.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- True
- False ✓

Question 5

1 pts

There are exactly two real values of c for which the distance between the vectors v and w in $\mathbf{R}^3$ written below is equal to 4.

$$v = \begin{pmatrix} 1 \\ -2c \\ 1 \end{pmatrix}, \quad w = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

$4 = \|v - w\| = \left\| \begin{pmatrix} 2 \\ -2(c-2) \\ 2 \end{pmatrix} \right\|$

$= 2\sqrt{1^2 + (c+1)^2 + 1^2}$

$= 2\sqrt{2 + (c+1)^2}$

$(c+1)^2 = 2 \Rightarrow c = \pm\sqrt{2} - 1$

- True ✓
- False

Question 6

1 pts

Suppose A is an $n \times n$ matrix and v is an eigenvector of A corresponding to the eigenvalue $\lambda = -2$.

Then $2v$ is an eigenvector of A corresponding to the eigenvalue $\lambda = -4$.

$Av = \lambda v$

$A(2v) = \lambda(2v)$

- True
- False ✓

Question 7

1 pts

Suppose W is a subspace of $\mathbf{R}^n$ and x is a vector in $\mathbf{R}^n$.

If the orthogonal projection of x onto W is the zero vector, then x is in $W^\perp$.

- True ✓
- False

Question 8

1 pts

Suppose A is an invertible 3×3 matrix. Then the product of the second row of A and the third column of A^{-1} must equal 0.

- True ✓
- False

Question 9

1 pts

If A is an invertible matrix and λ is an eigenvalue of A, then $\dfrac{1}{\lambda}$ is an eigenvalue of A^{-1}.

$Av = \lambda v$

$A^{-1}Av = \lambda A^{-1}v \Longrightarrow \frac{1}{\lambda}v = A^{-1}v$

$\lambda \neq 0$

since A invertible

- ✓ True
- False

Question 10

1 pts

Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the transformation given by $T(x, y, z) = (x - y - z,\ x - y - z)$.

2×3

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

Which of the following is true about T?

- ✓ T is neither one-to-one nor onto.
- T one-to-one but not onto.
- T is onto but not one-to-one.
- T is one-to-one and onto.

Question 11

1 pts

Which one of the following is a subspace of $\mathbf{R}^3$?

- ✓ The 1-eigenspace of $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 6 \end{pmatrix}$ span $\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right\}$
- $\text{Nul}\begin{pmatrix} 1 & 0 & 1 & 1 \\ -2 & 3 & -1 & 2 \\ 0 & 0 & 6 & -7 \end{pmatrix}$ $\in \mathbb{R}^4$

- $\text{Col}\begin{pmatrix} 1 & 0 & 1 \\ -2 & 3 & -1 \\ 0 & 0 & 6 \\ -1 & 1 & 0 \end{pmatrix}$ $\in \mathbb{R}^4$
- The set of all (x, y, z) in $\mathbf{R}^3$ satisfying $x - y - z = 1$.
- None of these are subspaces of $\mathbf{R}^3$

Question 12

1 pts

Consider the following matrix A and its reduced row echelon form:

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 0 & 1 \\ 1 & -2 & 3 \\ -2 & 4 & -8 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

x_2 free

Find a basis for $\mathrm{Nul}(A)$. $= \{x \mid Ax = 0\}$

- $\left\{ \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 3 \\ -8 \end{pmatrix} \right\}$

- $\left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \right\}$

Question 13

1 pts

$\Rightarrow \begin{pmatrix} 1 \\ 3 \end{pmatrix} = A\begin{pmatrix} -2 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ -3 \end{pmatrix} = A\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Suppose A is an invertible 2×2 matrix and $A^{-1} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Find the values a and b so that $A \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$.

a = -1

b = -3

Question 14

1 pts

Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation satisfying

$T \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $T \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

$A\begin{pmatrix} 2 & 4 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 3 & 0 \end{pmatrix}^{-1}$

Which one of the following statements is true?

$\Rightarrow$ span $\mathbb{R}^2$
$\Rightarrow$ onto
$\Rightarrow$ 2 pivots

- T is one-to-one and onto.

- ○ T is onto but not one-to-one.
- ○ T is one-to-one but not onto.
- ○ T is neither one-to-one nor onto

Question 15

1 pts

Which of the following transformations are linear? Select all that apply.

- ○ $T : \mathbf{R}^3 \to \mathbf{R}^3$ defined by $T(x, y, z) = (x, y + 1)$. ✗
- ○ $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(x, y) = (2x - \sin(y), \pi x)$. ✗
- ✓ ○ $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(x, y) = (x \ln(2), y)$.

Question 16

1 pts

Find the numbers a and b so that the following matrix is stochastic.

$$A = \begin{pmatrix} a & 1/3 \\ 1/8 & b \end{pmatrix}.$$

a = 7/8

b = 2/3

Enter each answer as a *fraction*.

Question 17 — 1 pts

Fill in the blanks:

$\underbrace{A \quad B}_{3 \times 4} \quad x$ $\quad 4\times 1$ $\quad x$ in $\mathbb{R}^k$

If A is a 3×7 matrix and B is a 7×4 matrix, then the transformation $T(x) = ABx$ has domain $\mathbf{R}^k$ and codomain $\mathbf{R}^p$,

$x \xrightarrow{AB} ABx$

where k = 4 and p = 3 .

Question 18 — 1 pts

Suppose A and B are 2×2 matrices satisfying $\det(A) = -4, \;\; \det(B) = 2.$

Find $\det(A^3 B^{-1})$ $= (\det A)^3 (\det B)^{-1}$

$(-4)^3 \; 2^{-1} = -32$

Question 19 — 1 pts

$Av = 3v \qquad Aw = -7w$

Suppose that v and w are eigenvectors of A with eigenvalues 3 and -7 respectively. Find $A(2v + 7w)$ in terms of v and w.

$A(2v + 7w) =$ 3×2 $v +$ -7×7 $w.$

$A2v = 3 \times 2v \qquad A7w = -7(7w)$

Question 20 — 1 pts

Let $A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 2 \end{pmatrix}$.

Which statement below is true?

$B-2I$ has 2 pivots 1 free

- A and B are both diagonalizable
- ✓ A is diagonalizable, but B is not diagonalizable
- B is diagonalizable, but A is not diagonalizable
- Neither A nor B is diagonalizable

Question 21

1 pts

Suppose $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 3.$

$R_1 \leftrightarrow R_3$, $(-2) \cdot R_3$, $R_1 \pm R_3$
$\cdot(-1)$ $\cdot(-2)$ $\cdot 1$

Find $\det \begin{pmatrix} g & h & i \\ d & e & f \\ g-2a & h-2b & i-2c \end{pmatrix}$.

$3 \cdot (-1) \cdot (-2) = 6$

Question 22

1 pts

Suppose that $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear transformation satisfying

$T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ and $T\begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Find $T\begin{pmatrix}2\\1\end{pmatrix}$.

$$\begin{pmatrix}2\\1\end{pmatrix} = \frac{1}{3}\left[\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}4\\0\end{pmatrix}\right]$$

$$T\begin{pmatrix}2\\1\end{pmatrix} = \frac{1}{3}\left[T\begin{pmatrix}\ \\ \ \end{pmatrix} + T\begin{pmatrix}\ \\ \ \end{pmatrix}\right]$$

$$= \frac{1}{3}\left[\begin{pmatrix}6\\4\end{pmatrix} + \begin{pmatrix}3\\-1\end{pmatrix}\right]$$

$$= \begin{pmatrix}3\\1\end{pmatrix}$$

- $\begin{pmatrix}3\\1\end{pmatrix}$
- $\begin{pmatrix}2\\3\end{pmatrix}$
- $\begin{pmatrix}9\\3\end{pmatrix}$
- $\begin{pmatrix}3\\5\end{pmatrix}$
- None of these
- $\begin{pmatrix}3\\2\end{pmatrix}$
- $\begin{pmatrix}3\\9\end{pmatrix}$
- $\begin{pmatrix}-3\\-5\end{pmatrix}$
- $\begin{pmatrix}-5\\-3\end{pmatrix}$

Question 23

1 pts

Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation of orthogonal projection onto the line $y = -x/4$.

Write the value of c so that $T\begin{pmatrix}8\\c\end{pmatrix} = \begin{pmatrix}8\\c\end{pmatrix}$.

$\begin{pmatrix}8\\c\end{pmatrix}$ on line $y=-x/4$

$c=-8/4$

-2

Question 24

1 pts

Let $B = \begin{pmatrix}7 & -8\\4 & -5\end{pmatrix}$. Write the eigenvalues of B in increasing order.

The smaller eigenvalue is -1, and the larger eigenvalue is 3.

$(\lambda-7)(\lambda+5)$

$(7-\lambda)(-5-\lambda)+32=0$

$\lambda^2-2\lambda-3=0$

$(\lambda-3)(\lambda+1)=0$

Question 25

1 pts

Let $T:\mathbf{R}^3 \to \mathbf{R}^3$ be the transformation that reflects vectors across the yz-plane, and let A be the standard matrix for T.

Which one of the following statements is true?

$A\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}-1\\0\\0\end{pmatrix} \Rightarrow \lambda=-1$

$A\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} \Rightarrow \lambda=1$

$A\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} \Rightarrow \lambda=1$

- ✓ $\dim(\mathrm{Nul}(A-I)) = 2$ and $\dim(\mathrm{Nul}(A+I)) = 1$
- $\dim(\mathrm{Nul}(A-I)) = 1$ and $\dim(\mathrm{Nul}(A+I)) = 2$
- $\dim(\mathrm{Nul}(A-I)) = 1$ and $\dim(\mathrm{Nul}(A+I)) = 1$
- $\dim(\mathrm{Nul}(A)) = 2$ and $\dim(\mathrm{Nul}(A+I)) = 1$
- $\dim(\mathrm{Nul}(A)) = 1$ and $\dim(\mathrm{Nul}(A+I)) = 2$

- None of these

Question 26

1 pts

Select the matrix below whose 7-eigenspace is a line.

1 free

$A - 7I$

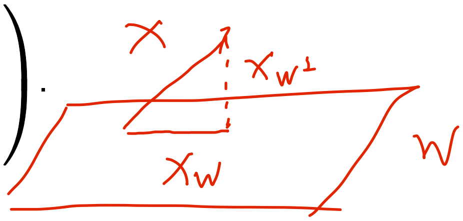
- $\begin{pmatrix} 7 & 0 & 0 \\ 1 & 7 & 0 \\ 0 & 1 & 7 \end{pmatrix}$
- $\begin{pmatrix} 7 & 1 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$
- $\begin{pmatrix} 7 & 0 & 1 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$
- $\begin{pmatrix} 7 & 0 & 1 \\ 0 & 7 & 1 \\ 0 & 0 & 7 \end{pmatrix}$
- $\begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$
- $\begin{pmatrix} 7 & 0 & 0 \\ 1 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$

Question 27

1 pts

Suppose W is a subspace of $\mathbf{R}^n$ and x is a vector in $\mathbf{R}^n$ satisfying

$$x_W = \begin{pmatrix} 7 \\ 0 \\ 0 \end{pmatrix} \text{ and } x_{W^\perp} = \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix}.$$



What is the distance from x to W?

$\|x_{W^\perp}\| = \sqrt{0^2 + 3^2 + (-4)^2} = 5$

5

Question 28

1 pts

$u \cdot v = 0$

Suppose u and v are *orthogonal* vectors in $\mathbf{R}^4$ satisfying $||u|| = 2$ and $||v|| = 3$.

Calculate the dot product $(3u - v) \cdot (2v)$ $= 6u \cdot v - 2v \cdot v$

$= 0 - 2 \cdot \|v\|^2$

$= 2 \cdot 3^2$

18

Question 29

1 pts

Suppose A is a 3×2 matrix, b is a vector, and $\hat{x} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ is a least-squares solution to $Ax = b$.

Which one of the following statements must be true?

not $\hat{x}$, should be $A\hat{x}$

- $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ is the orthogonal projection b onto $\mathrm{Col}(A)$.

○ $\text{rank}(A) = 2$

○ $A \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ is the closest vector to b in $\text{Col}(A)$.

○ The closest vector to b in $\text{Col}(A)$ is $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

Question 30

1 pts

$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

Let W be the line $x_2 = -3x_1$ in $\mathbf{R}^2$. Find the number a so that the orthogonal projection of $\begin{pmatrix} a \\ 1 \end{pmatrix}$ onto W is the zero vector.

$\perp W$

$\begin{pmatrix} 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} a \\ 1 \end{pmatrix} = 0$

$a = 3$

3

Question 31

1 pts

Suppose a positive stochastic matrix A has 1-eigenspace equal to the span of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Let $v = \begin{pmatrix} 50 \\ 10 \end{pmatrix}$. As n gets large, $A^n v$ approaches some vector $\begin{pmatrix} a \\ b \end{pmatrix}$. Find a and b.

$A\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$a =$ 40

$A^n\begin{pmatrix} 50 \\ 10 \end{pmatrix} = 60A^n\begin{pmatrix} 5/6 \\ 1/6 \end{pmatrix}$

$b =$ 20 .

$60A^n\begin{pmatrix} 5/6 \\ 1/6 \end{pmatrix} \rightarrow 60\begin{pmatrix} 2/3 \\ 1/3 \end{pmatrix}$

Question 32

1 pts

Let $W = \text{Span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$.

Find the dimension of W.

2 pivots

2

Question 33

1 pts

Select the matrix A that satisfies

$$A\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}, \quad A\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}, \quad A\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}.$$

$\lambda = -1$ $\lambda = 2$ $\lambda = -3$

- ✓ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}^{-1}$
- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & -3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}^{-1}$
- $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}^{-1}$

- ○ $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$
- ○ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$
- ○ None of these

Question 34

1 pts

Find the value of h so that the following set of vectors is linearly dependent.

$$\left\{ \begin{pmatrix} h \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

do row reduction
2 pivots

3

Question 35

1 pts

Suppose $T : \mathbf{R}^m \to \mathbf{R}^n$ and $U : \mathbf{R}^n \to \mathbf{R}^m$ are linear transformations.

Which one of the following statements *must* be true?

$U \circ T$: $\mathbb{R}^m \xrightarrow{T} \mathbb{R}^n \xrightarrow{U} \mathbb{R}^m$, $x \to Tx \to U(Tx) = U\circ T(x)$

- ✓ ○ The transformation $T \circ U$ has domain $\mathbf{R}^n$ and codomain $\mathbf{R}^n$.

$T \circ U$: $\mathbb{R}^n \xrightarrow{U} \mathbb{R}^m \xrightarrow{T} \mathbb{R}^n$, $x \to Ux \to T(Ux)$

- ○ If $m > n$, then $T \circ U$ is not onto.

No $U: [1\ 0]$, $T: \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

○ U is one-to-one if for every x in $\mathbf{R}^n$, there is a y in $\mathbf{R}^m$ so that $U(x) = y$.

Question 36

1 pts

Some *diagonalizable* 6×6 matrix A has characteristic polynomial

$\det(A - \lambda I) = (4-\lambda)^3 \lambda^2 (-3-\lambda)$.

$\lambda = 0$ algebraic Multiplicity $= 2$.

$A - 0I$

What is the dimension of the null space of A?

- ○ 2
- ○ 1
- ○ 3
- ○ 4
- ○ 5
- ○ 6
- ○ Not enough information to know the answer

Question 37

1 pts

$A_{m \times n}$

Which one of the following matrices satisfies both of the following conditions?

1. $\mathrm{Col}(A)$ is a subspace of $\mathbf{R}^4$. $m = 4$ 4 rows
2. $\mathrm{Nul}(A)$ is the line $x_2 = 8x_1$ in $\mathbf{R}^2$. $n = 2$

$\begin{bmatrix} -8 & 1 \\ \vdots & \vdots \end{bmatrix}$ $-8x_1 + x_2 = 0$

○

$$A = \begin{pmatrix} -8 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

- $A = \begin{pmatrix} 1 & 8 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

- $A = \begin{pmatrix} -8 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

- $A = \begin{pmatrix} 8 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

- $A = \begin{pmatrix} 8 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

- $A = \begin{pmatrix} 1 & -8 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

- none of these

Question 38

1 pts

Let W be subspace of $\mathbf{R}^3$ consisting of all (x, y, z) satisfying $x = y = z$.

$\begin{cases} x = z \\ y = z \end{cases}$ $\text{span}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

What is the dimension of $W^{\perp}$?

$= \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\}$

2

Question 39

1 pts

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be reflection across the y-axis, let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be rotation counterclockwise by 90 degrees, and let $V : \mathbf{R}^2 \to \mathbf{R}^2$

be reflection across the line $y = -x$. $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Which one of the following transformations has standard matrix equal to

$$\underbrace{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}_{T}\underbrace{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}}_{V}?$$

- ○ $T \circ U$
- ○ $U \circ T$
- ○ $T \circ V$ ✓
- ○ $V \circ T$
- ○ $U \circ V$
- ○ $V \circ U$

Question 40

1 pts

Let $A = \begin{pmatrix} -1 & 2 \\ 1 & 1 \\ 2 & -1 \end{pmatrix}$.

Find the value of a so that $(\mathrm{Col}\ A)^{\perp} = \mathrm{Span}\left\{\begin{pmatrix} a \\ -1 \\ 1 \end{pmatrix}\right\}$.

Must satisfy ① & ②

① $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} a \\ -1 \\ 1 \end{pmatrix} = 0 \Rightarrow a = 1$

&

② $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a \\ -1 \\ 1 \end{pmatrix} = 0 \Rightarrow a = 0$

does not exist

Question 41

1 pts

Suppose A is the matrix that implements reflection across the line $y = -4x$ in $\mathbf{R}^2$. Which of the following statements is true?

- A is invertible and diagonalizable.
- A is invertible but is not diagonalizable.
- A is diagonalizable but not invertible.
- A is neither invertible nor diagonalizable.

Question 42

1 pts

Let W be the subspace of $\mathbf{R}^3$ consisting of all (x, y, z) in $\mathbf{R}^3$ satisfying $x - 3y - z = 0$, and let B be the matrix for orthogonal projection onto W.

2 free ⇒ plane

Which one of the following statements must be true? If none must be true, select "none of these" as your answer.

W

- $\mathrm{rank}(B) = 1$

$\dim(W) = \mathrm{rank}(B) = 2.$

-

$$B\begin{pmatrix}4\\1\\1\end{pmatrix}=\begin{pmatrix}4\\1\\1\end{pmatrix}$$

$\begin{pmatrix}4\\1\\1\end{pmatrix}$ in W

- ○ $W^\perp$ is a plane in $\mathbf{R}^3$ ✗ line span$\left\{\begin{pmatrix}1\\-3\\-1\end{pmatrix}\right\}$
- ○ The 1-eigenspace of B is 1-dimensional. ✗ 2-dim
- ○ None of these

Question 43

1 pts

Suppose A is a 4 x 4 matrix whose entries are real numbers, and suppose $1+i$ and $2-i$ are eigenvalues of A.

$1-i$

$2+i$

Which statement below must be true? If none must be true, select "none of these" as your answer.

- ○ A has no real eigenvalues. ✓
- ○ $\det(A)=0$ ✗ not inverible ⇒ has $\lambda=0$
- ○ The equation $Ax=b$ is inconsistent for some b in $\mathbf{R}^4$ not invertible
- ○ A is a positive stochastic matrix. ✗
- ○ none of these

Question 44

1 pts

Which of the following gives least-squares line $y=Cx+D$ for the data points $(0,1)$, $(2,-2)$, and $(3,2)$?

$$A=\begin{pmatrix}0&1\\2&1\\3&1\end{pmatrix},\quad \begin{pmatrix}1\\-2\\2\end{pmatrix}$$

x 1 y

$A\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$A^T A \begin{pmatrix} C \\ D \end{pmatrix} = A^T \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

○ We get C and D by solving

$$\begin{pmatrix} 0 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 0 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

○ We get C and D by solving

$$\begin{pmatrix} 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 0 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

○ We get C and D by solving

$$\begin{pmatrix} 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix}$$

○ We directly solve

$$\begin{pmatrix} 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

○ We get C and D by solving

$$\begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

○ We get C and D by solving

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

Question 45 — 1 pts

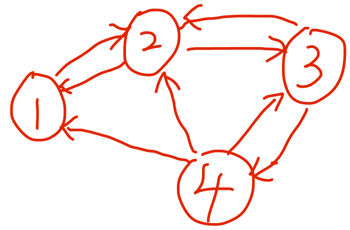
Consider an internet with 4 pages.

Page 1 links only to page 2.

Page 2 links to pages 1 and 3.

Page 3 links to pages 2 and 4.

Page 4 links to pages 1, 2, and 3.



What is the importance matrix (also known as Google matrix) for this internet?

From

✓ To

$$A = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 0 & 1/2 & 0 & 1/3 \\ 1 & 0 & 1/2 & 1/3 \\ 0 & 1/2 & 0 & 1/3 \\ 0 & 0 & 1/2 & 0 \end{pmatrix} \end{array}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 \\ 0 & 1/2 & 1 & 1/2 \\ 1/3 & 1/3 & 1/3 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1/2 & 0 & 1/3 \\ 1 & 1 & 1/2 & 1/3 \\ 0 & 1/2 & 1 & 1/3 \\ 0 & 0 & 1/2 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1/2 & 1/3 & 0 & 1/4 \\ 1/2 & 1/3 & 1/3 & 1/4 \\ 0 & 1/3 & 1/3 & 1/4 \\ 0 & 0 & 1/3 & 1/4 \end{pmatrix}$$

Question 46

1 pts

Let T be a linear transformation such that $T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\\-1\end{pmatrix}$ and $T\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}0\\-1\\-2\end{pmatrix}$. Then, $T\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$, where $a =$ [1], $b =$ [-1] and $c =$ [-3].

$T\begin{pmatrix}2\\0\end{pmatrix} = T\left[\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}1\\-1\end{pmatrix}\right]$

Question 47

1 pts

What is the determinant of $\begin{pmatrix}1 & 0 & -1 & 0 & 0 & 15\\ 0 & 7 & 5 & 0 & 19 & 0\\ 0 & 0 & -2 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0 & 0 & -1\end{pmatrix}$?

$1 \cdot 7 \cdot (-2) \cdot (1) \cdot (-1) \cdot (-1) = -14$

Question 48

1 pts

Suppose that A is a 6x7 matrix and B is a 7x6 matrix such that AB is the 6x6 identity matrix. Is BA equal to the 7x7 identity matrix? [Select]

$\mathbb{R}^7 \xrightarrow{A} \mathbb{R}^6 \xrightarrow{B} \mathbb{R}^7$

No, Never happens.
there must be $x \neq y$
$Ax = Ay$

Question 49

1 pts

Let S be a region in the plane with area 3. Let T be the linear transformation $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2)$. What is the area of $T(S)$? $= |-2| \cdot 3$

$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $\det(A) = -2$

6

Question 50

1 pts

You are tasked with uncovering information about a secret matrix, some of whose entries are hidden:

$\boxed{?}_1$ or $\boxed{?}_2 = 0$

$$A = \begin{pmatrix} -1 & 0 & \boxed{?}_2 \\ ? & 4 & ? \\ \boxed{?}_1 & 0 & 0 \end{pmatrix}$$

$\det A = \boxed{?}_1 \cdot (-1)^{3+1} \cdot \underbrace{\det\begin{pmatrix} 0 & \boxed{?}_2 \\ 4 & ? \end{pmatrix}}_{0 - 4\boxed{?}_2}$

(the "?" represent hidden entries). Given that $\mathrm{rank}(A) = 2$, how many distinct eigenvalues does A have?

If $\boxed{?}_1 = 0$ then $\lambda_1 = 0$, $\begin{bmatrix} -1-\lambda & 0 \\ ? & 4-\lambda \end{bmatrix}$ will provide $\lambda_2 = -1$, $\lambda_3 = 4$

If $\boxed{?}_2 = 0$ then $\lambda_1 = -1$, $\begin{bmatrix} 4-\lambda & ? \\ 0 & -\lambda \end{bmatrix}$ will provide $\lambda_2 = 4$, $\lambda_3 = 0$.

- 0
- 1
- 2
- 3 ✓

Quiz saved at 7:57pm

Submit Quiz