Math 1553 Quiz 1

Solutions

Note: As the quiz was administered on Canvas, some values in the problems may differ from student to student.

1. Find the values of h that make the following system of equations consistent:

$$x + y - z = 5$$

$$3x + 3y - 3z = h$$

Solution: Putting the system into an augmented matrix, we get:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 3 & 3 & -3 & h \end{array}\right)$$

Subtracting 3 times the first row from the second row gives us:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 0 & 0 & h-15 \end{array}\right)$$

Because the second row is a row of zeroes, the system can only be consistent if $h - 15 = 0$. Therefore $h = 15$.

2. If a system of linear equations has 5 equations and 7 variables, then it must have infinitely many solutions.

Solution: False. It is possible for such a system to have no solution. An example could be:

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \end{array}\right)$$

Which row reduces to:

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{array}\right)$$

And this is clearly an inconsistent system.

3. Recall that the general equation for a circle is:

$$A(x^2 + y^2) + Bx + Cy + D = 0$$

Find an equation for the circle passing through the points $(0, 0)$, $(1, 0)$, and $(0, 4)$, where $A = 1$.

Your answer will be in the form $(x^2 + y^2) - x + Cy = 0$. What is C?

Solution: Plugging in the three points, given $A = 1$ gives us

(0,0) $\implies D = 0$

(1,0) $\implies 1 + B + D = 0 \implies B + D = -1$

(0,4) $\implies 16 + 4C + D = 0 \implies 4C + D = -16$

Since A is known, our unknowns are B, C, and D. So as a matrix:

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 4 & 1 & -16 \end{array}\right)$$

Which can be row reduced to:

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -4 \end{array}\right)$$

Giving us $B = -1$, $C = -4$, and $D = 0$.

Thus $C = -4$.

4. Suppose we have two linear equations in three variables. Which of the following are possibilities for the set of solutions to this system? Select all that apply.

Solution:

☐ One point in $\mathbb{R}^3$

☑ A plane in $\mathbb{R}^3$

☑ No solution

☑ A line in $\mathbb{R}^3$

Since there are more variables than equations, the system is forced to admit at least one free variable. So if a solution exists, it CANNOT be unique. Therefore one point in $\mathbb{R}^3$ is not a possibility.

Geometrically, a linear equation in three variables represents a plane in $\mathbb{R}^3$.

If both linear equations describe the same plane, then the plane intersects itself everywhere. So a plane in $\mathbb{R}^3$ is a possibility.

Of course, it is also possible for two planes $\mathbb{R}^3$ to not intersect at all. So no solution is a possibility.

If two (different) planes in $\mathbb{R}^3$ do intersect, they must form a line. So a line in $\mathbb{R}^3$ is a possibility.

5. Consider the following system of linear equations:

$$x + y + z = 3$$

$$x + y - z = 1$$

$$2x + y + 3z = 6$$

Which of the following points in $\mathbb{R}^3$ is a solution?

Solution:

☐ $(3, 0, 0)$

☐ $(0, 0, 0)$

☑ $(1, 1, 1)$

☐ $(2, 0, 1)$

Putting the equations in an augmented matrix gives us:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 1 & -1 & 1 \\ 2 & 1 & 3 & 6 \end{array}\right)$$

And reducing to reduced row echelon form gives us:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

Since there is a pivot in every column, there are no free variables and thus $(1, 1, 1)$ is the unique solution.