Quiz 6 solution

2. F a) Every $T: \mathbb{R}^{10} \to \mathbb{R}^1$ is onto

T b) $T: \mathbb{R}^4 \to \mathbb{R}^4$ is onto then it must be 1-1

Ans: a) False take $T = [0\ 0 \cdots 0]$

b) True T has 4 pivots

3. $T: \mathbb{R}^3 \to \mathbb{R}^3$ $T(e_1) = e_2 + e_3$

$T(e_2) = e_1 + e_3$

$T(e_3) = e_1$

Matrix for T :

Ans: $\hookrightarrow \left[T(e_1)\ T(e_2)\ T(e_3) \right] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

4. $A_{5\times 3}$, $\text{rank}(A) = 2$, $T(v) = Av$ Is T 1-1 ?

Ans: No

$\text{rank}(A) = 2 \Rightarrow$ A has 2 pivots $\Rightarrow$ T is not 1-1

(A has $3 - 2 = 1$ free variable)

($Ax = 0$ has ∞ solutions)

5. $T: \mathbb{R}^2 \to \mathbb{R}^3$ Range(T) is a line

Ans: $T(\vec{v}) = \underset{3\times 2}{A}\ \underset{2\times 1}{\vec{v}}$

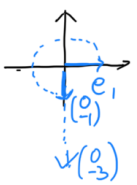
Range(T) is a line $\Leftrightarrow \dim(\text{Col}(A)) = 1$

$\Leftrightarrow$ 1 pivots

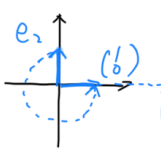
$\Rightarrow$ T is not 1-1, T is not onto

6. $T: \mathbb{R}^2 \to \mathbb{R}^2$ $\underbrace{\text{rotate } \frac{3\pi}{2}}_{R}$, $\underbrace{\text{scaling by 3}}_{S}$.

Ans:



$e_1 \xrightarrow{R} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 0 \\ -3 \end{pmatrix}$



$e_2 \xrightarrow{R} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$T(e_1) = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$, $T(e_2) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Matrix for $T = \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$