Q2:

Consider the matrix

$$A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

Fill in the missing entries in the matrix of cofactors of A:

$$\begin{pmatrix} 4 & a & b \\ c & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Recall that the entry in the ith row and jth column of the cofactor matrix is $(-1)^{i+j}\det A_{ij}$.

Solution:

For each value a, b, and c, you want to apply the formula given in the question

$(-1)^{i+j}$ is either 1 or -1 depending on the position within the matrix

$\det A_{ij}$ is the determinant of the submatrix formed when you eliminate the row and column corresponding to that position (ith row, jth column)

a = -1 *(-1*2-0*0) = 2

b = 1* (-1*1-2*0) = -1

c = -1*(1*2-(-1)*1)= -3

Q3

Suppose that A is a 3×3 matrix, that the cofactor matrix of A is

$$\begin{pmatrix} -1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$$

and that $\det(A) = -1$. Find the inverse of A.

Solution

In this problem you want to apply the cofactor method of computing an inverse matrix

$$A^{-1} = \frac{1}{\det A} * C^T$$

In the Question, You are given C, the matrix of cofactors, as well as Det(A)

C^T is the transpose matrix of C

C:

-1	1	-1
1	-1	-1
1	1	-1

C^T

-1	1	1
1	-1	1
-1	-1	-1

$A^{-1} = \frac{C^T}{\det A} = \frac{C^T}{-1}$

1	-1	-1
-1	1	-1
1	1	1

Q4

Consider the matrix

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Which of the following vectors is an eigenvector for A? *Select all that apply.*

- [ ] $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- [ ] $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- [ ] $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Solution:

To check if a vector is an eigenvector, just do the matrix multiplication A*v and if the result is a multiple of the original vector, that vector is an eigenvector

i. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ - not an eigenvector

ii. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ - not an eigenvector

iii. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ - is an eigenvector with $\lambda = 1$

iv. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ - is an eigenvector with $\lambda = -1$

v. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ - never an eigenvector, because eigenvectors are not allowed be the zero vector

Q5

Let $A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$. Which of the following are eigenvalues of A with a 1-dimensional eigenspace? *Select all that apply.*

- ☐ 0
- ☐ 2
- ☐ 4

Solution

The eigenvalues are the roots of the characteristic polynomial $f(\lambda) = \det(A - \lambda I)$

In this case $f(\lambda) = (1 - \lambda) * (1 - \lambda) - 3 * 3 = \lambda^2 - 2\lambda - 8$

Which has roots $\lambda = 4, -2$

This rules out the first two choices of 0 and 2

The question also asks that the eigenvalue has a 1-dimensional eigenspace. The λ −eigenspace is just the null space of $A - \lambda I$, and its dimension is the nullity, or number of non-pivot columns in $A - \lambda I$

$A - \lambda I = \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \rightarrow (Row\ reduction) \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

Since there is 1 non-pivot column, the 4-eigenspace is a line.

Q6

Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation that reflects about the line $y = 2x$ in $\mathbb{R}^2$. What are the eigenvalues of the standard matrix for T? *Select all that apply.*

- ☐ 0
- ☐ 1
- ☐ -1
- ☐ 2
- ☐ -2
- ☐ There are no eigenvalues

Solution:

- The green line is the line y=2x
- The blue vector is an eigenvector with eigenvalue 1, because when it is reflected across the green line, the result is $1 * V_1$
- The red vector is an eigenvector with eigenvalue -1, because when it is reflected across the green line, the result is $-1 * V_2$

For those who are color-blind:

Green = L
Blue = V_1
Red = V_2