Announcements Aug 19

- Mathematical autobiography due on Friday Teams channel → Canvas → Assignm...
- WeBWorK Warmup due Thursday night (not for a grade) or maybe Sunday
- Warmup Quiz Friday 8 am - 8 pm EDT Canvas → Quizzes, not for grade (← not for grade)
- Fill out survey for office hours in When2Meet (link sent in Teams and email)
- Office hours for this week only: Thu 4:15-5:15 EDT
- Studio on Friday (check your email for code/link)

Midterm 2 "correction" Oct 16

- Canvas → Master Web Site for lots of stuff

Section 1.1

Solving systems of equations

Outline of Section 1.1

- Learn what it means to solve a system of linear equations
- Describe the solutions as points in $\mathbb{R}^n$
- Learn what it means for a system of linear equations to be inconsistent

Solving equations

Solving equations

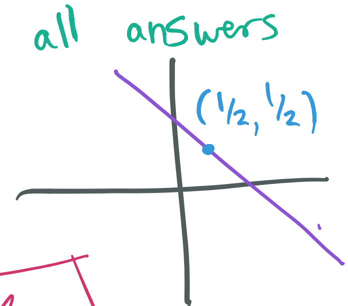
What does it mean to solve an equation?

$2x = 10$

x = 5

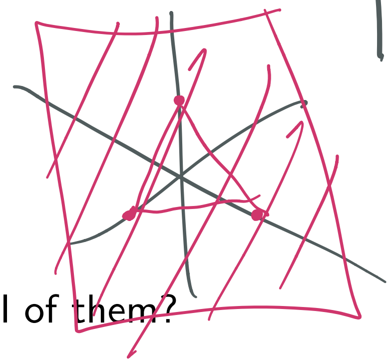
$x + y = 1$

one answer: x = 0, y = 1 or (0, 1) or....



$x + y + z = 1$

one answer: (1, 0, 0)



Find one solution to each. Can you find all of them?

A solution is a *list* of numbers. For example $(3, -4, 1)$.

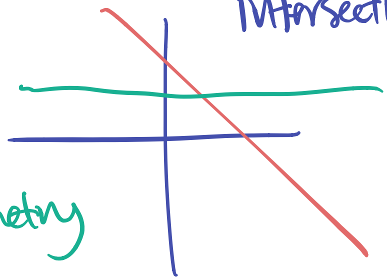
Solving equations

What does it mean to solve a system of equations?

intersection

$$x + y = 2$$
$$y = 1$$

finding x, y that work for both eqns.



geometry

algebra

What about...

pt

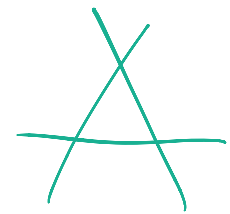
plane?

line?

nothing?

$$x + y + z = 3$$
$$x + y - z = 1$$
$$x - y + z = 1$$

intersection of 3 planes?



Is $(1, 1, 1)$ a solution? Is $(2, 0, 1)$ a solution? What are all the solutions?

yes

no

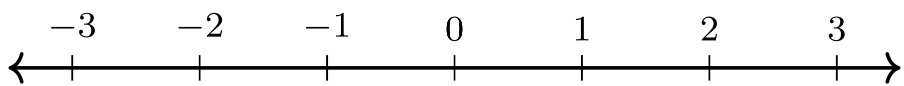
Soon, you will be able to see just by looking that there is exactly one solution.

$$\mathbb{R}^n$$

$\mathbb{R}^n$

$\mathbb{R}$ = denotes the set of all real numbers $\mathbb{R}^1$

Geometrically, this is the *number line*.



$\mathbb{R}^n$ = all ordered n-tuples (or lists) of real numbers $(x_1, x_2, x_3, \ldots, x_n)$

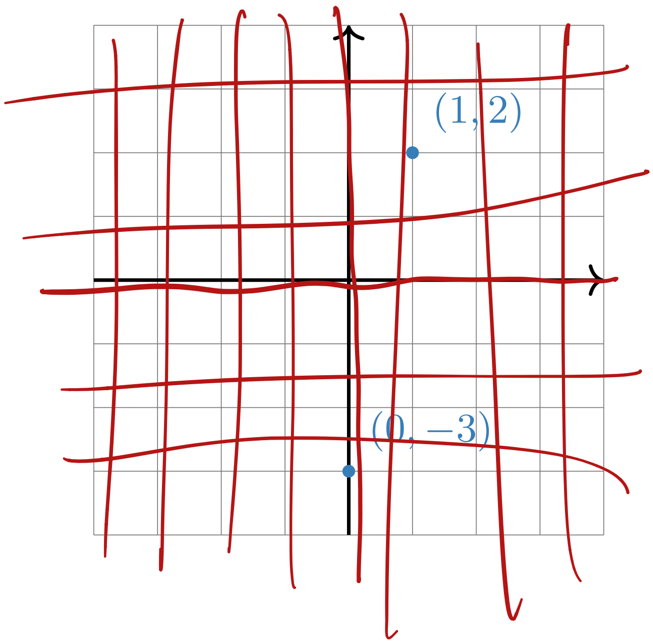
Solutions to systems of equations are exactly points in $\mathbb{R}^n$.

A Point in $\mathbb{R}^4$: $(2, 3, 4, 7)$

or $(\pi, \sqrt{2}, -e, 0)$

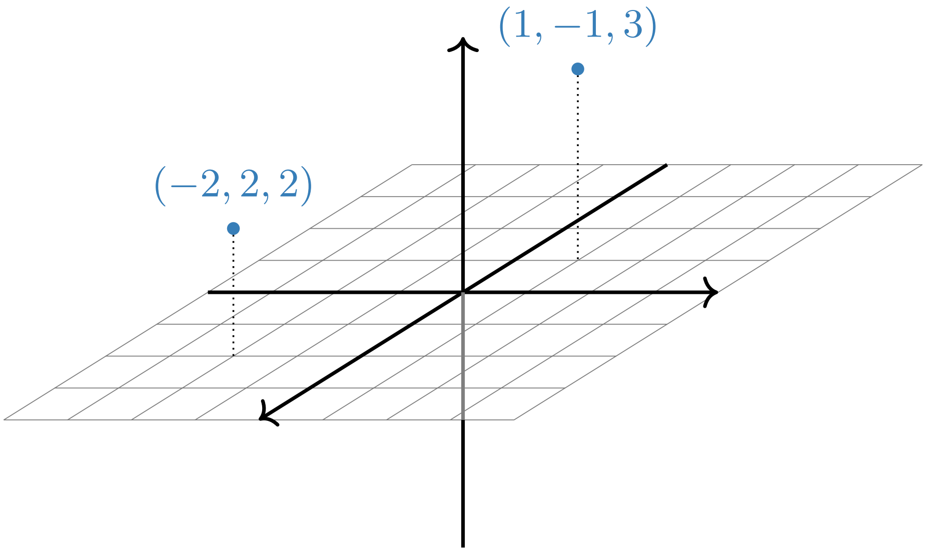
$\mathbb{R}^n$

When $n = 2$, we can visualize of $\mathbb{R}^2$ as the *plane*.

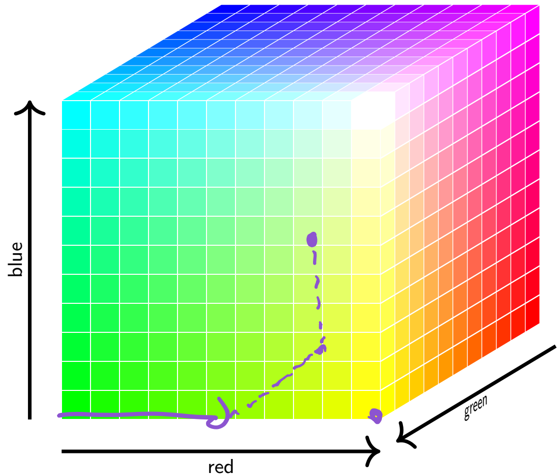


$\mathbb{R}^n$

When $n = 3$, we can visualize $\mathbb{R}^3$ as the *space* we (appear to) live in.



We can think of the space of all *colors* as (a subset of) $\mathbb{R}^3$:

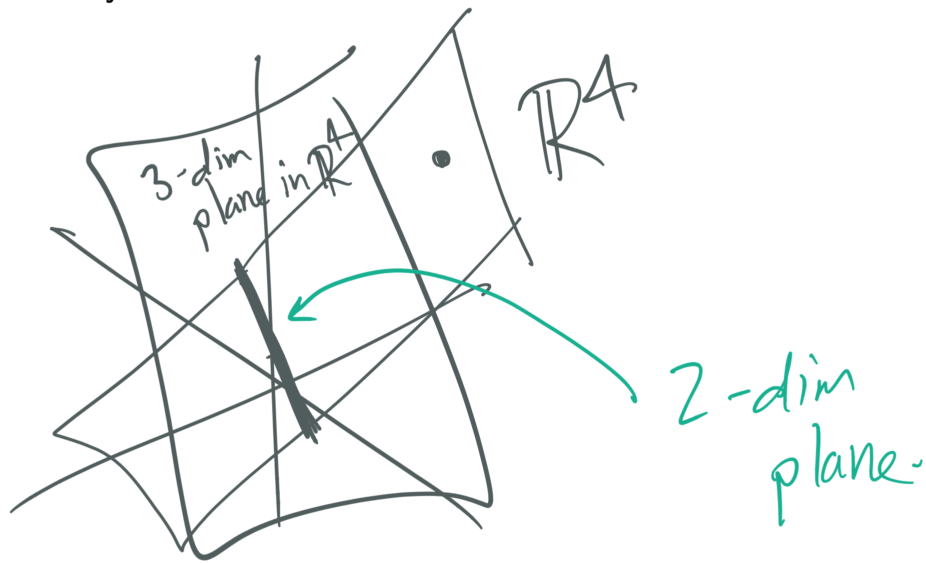


So what is $\mathbb{R}^4$? or $\mathbb{R}^5$? or $\mathbb{R}^n$?

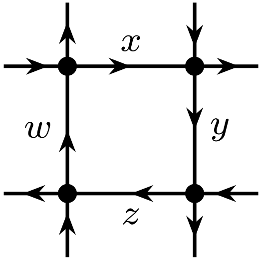
...go back to the *definition*: ordered n-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They're still "geometric" spaces, in the sense that our intuition for $\mathbb{R}^2$ and $\mathbb{R}^3$ sometimes extends to $\mathbb{R}^n$, but they're harder to visualize.



Last time we could have used $\mathbb{R}^4$ to label the amount of traffic (x, y, z, w) passing through four streets.



We'll make definitions and state theorems that apply to any $\mathbb{R}^n$, but we'll only draw pictures in $\mathbb{R}^2$ and $\mathbb{R}^3$.

$\mathbb{R}^n$ and QR codes

This is a 21×21 QR code. We can also think of this as an element of $\mathbb{R}^n$.

How? Which n? $n = 21^2$

What about a greyscale image?

This is a powerful idea: instead of thinking of a QR code as 441 pieces of information, we think of it as one piece of information.

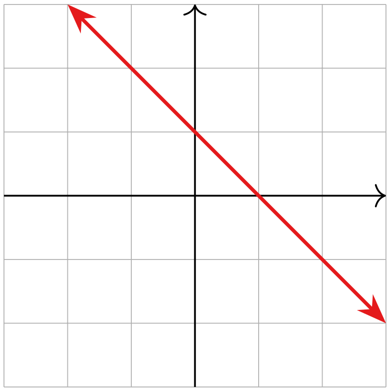
Visualizing solutions: a preview

One Linear Equation

What does the solution set of a linear equation look like?

$x + y = 1 \rightsquigarrow$ a line in the plane: $y = 1 - x$

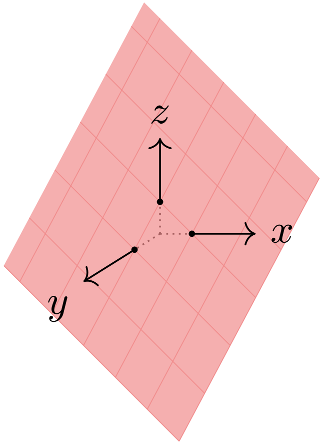
$\mathbb{R}^2$



One Linear Equation

What does the solution set of a linear equation look like?

$x + y + z = 1 \rightsquigarrow$ a plane in space:



z=0 has

solution set: xy-plane

One Linear Equation

Continued

What does the solution set of a linear equation look like?

$x + y + z + w = 1 \rightsquigarrow$ a "3-plane" in "4-space"...

One eqn in 100 vars

⇝ Soln set is 99-dim'l plane in $\mathbb{R}^{100}$

Two eqns in 100 vars: intersect two of those.

Systems of Linear Equations

What does the solution set of a *system* of more than one linear equation look like?

$$x - 3y = -3$$
$$2x + y = 8$$

intersect 2 lines

What are the other possibilities for two equations with two variables?

each eqn in n vars is (n-1)-dim plane in $\mathbb{R}^n$ (prev. slide)

What if there are more variables? More equations?

n variables

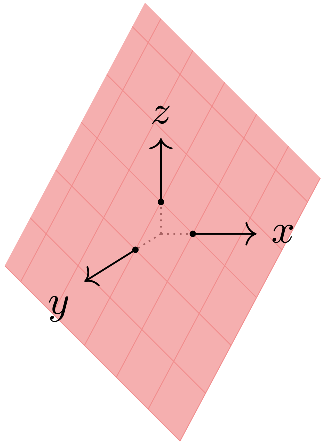
k eqns

→ intersect k (n-1)-dim plane in $\mathbb{R}^n$

Poll

Is the plane in $\mathbb{R}^3$ from the previous example equal to $\mathbb{R}^2$? What about the xy-plane in $\mathbb{R}^3$?

1. yes + yes
2. yes + no
3. no + yes
4. no + no



Consistent versus Inconsistent

We say that a system of linear equations is consistent if it has a solution and inconsistent otherwise.

$$x + y = 1$$
$$x + y = 2$$

Why is this inconsistent?

no x & y work for both

alg.

parallel lines

geometry

What are other examples of inconsistent systems of linear equations?

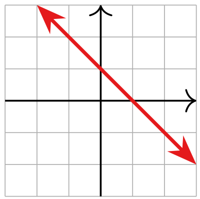
variables x, y, z

z = 0
z = 1

xy-plane parallel to

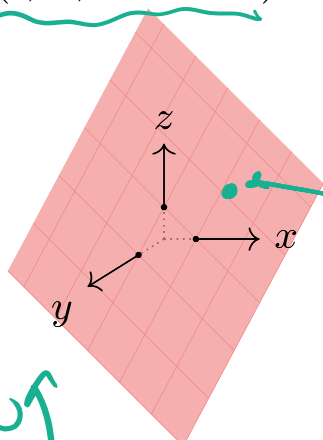
Parametric form

The equation $y = 1 - x$ is an implicit equation for the line in the picture.



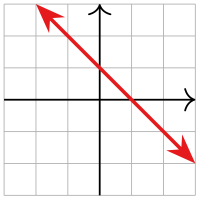
It also has a parametric form: $(t, 1-t)$

Similarly the equation $x + y + z = 1$ is an implicit equation. One parametric form is: $(t, w, 1-t-w)$.



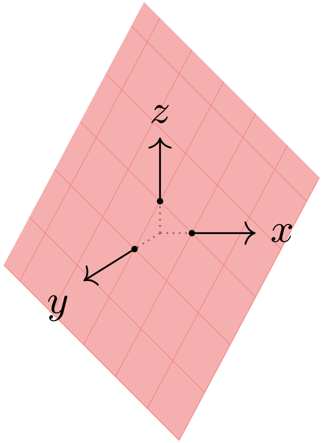
Parametric form

The equation $y = 1 - x$ is an implicit equation for the line in the picture.



It also has a parametric form: $(t, 1 - t)$

Similarly the equation $x + y + z = 1$ is an implicit equation. One parametric form is: $(t, w, 1 - t - w)$.



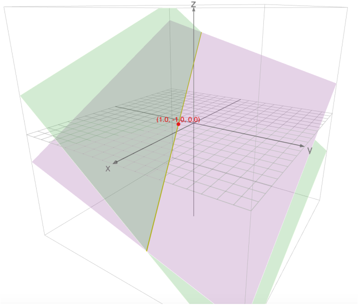
What is an implicit equation and a parametric form for the xy-plane in $\mathbb{R}^3$?

Parametric form

The system of equations

$$\begin{aligned} 2x + y + 12z &= 1 \\ x + 2y + 9z &= -1 \end{aligned}$$

is the implicit form for the line of intersection in the picture.



The line of intersection also has a parametric form: $(1 - 5z, -1 - 2z, z)$

We think of the former as being the problem and the latter as being the explicit solution. One of our first tasks this semester is to learn how to go from the implicit form to the parametric form.

Summary of Section 1.1

- A solution to a system of linear equations in n variables is a point in $\mathbb{R}^n$.
- The set of all solutions to a single equation in n variables is an $(n-1)$-dimensional plane in $\mathbb{R}^n$
- The set of solutions to a system of m linear equations in n variables is the intersection of m of these $(n-1)$-dimensional planes in $\mathbb{R}^n$.
- A system of equations with no solutions is said to be inconsistent.
- Line and planes have implicit equations and parametric forms.

Section 1.2

Row reduction

Outline of Section 1.2

- Solve systems of linear equations via elimination
- Solve systems of linear equations via matrices and row reduction
- Learn about row echelon form and reduced row echelon form of a matrix
- Learn the algorithm for finding the (reduced) row echelon form of a matrix
- Determine from the row echelon form of a matrix if the corresponding system of linear equations is consistent or not.

Solving systems of linear equations by elimination

Example

Solve:

$$-y + 8z = 10$$
$$5y + 10z = 0$$

How many ways can you do it?

Example

Solve:

$$
\begin{aligned}
-x + y + 3z &= -2 \\
2x - 3y + 2z &= 14 \\
3x + 2y + z &= 6
\end{aligned}
$$

Hint: Eliminate x!

Solving systems of linear equations with matrices

Example

Solve:

$$
\begin{aligned}
-y + 8z &= 10 \\
5y + 10z &= 0
\end{aligned}
$$

It is redundant to write x and y again and again, so we rewrite using (augmented) *matrices*. In other words, just keep track of the coefficients, drop the $+$ and $=$ signs. We put a vertical line where the equals sign is.

$$
\left(\begin{array}{rr|r} -1 & 8 & 10 \\ 5 & 10 & 0 \end{array}\right) \rightsquigarrow
$$

Example

Solve:

$$\begin{aligned} -x + y + 3z &= -2 \\ 2x - 3y + 2z &= 14 \\ 3x + 2y + z &= 6 \end{aligned}$$

Again we rewrite using augmented matrices...

Row operations

Our manipulations of matrices are called row operations:

row swap, row scale, row replacement

If two matrices differ by a sequence of these three row operations, we say they are row equivalent.

Goal: Produce a system of equations like:

$$\begin{aligned} x \qquad\qquad &= 2 \\ y \qquad &= 1 \\ z &= 5 \end{aligned}$$

What does this look like in matrix form?

Row operations

Why do row operations not change the solution?

Solve:

$$\begin{aligned} x + y &= 2 \\ -2x + y &= -1 \end{aligned}$$

System has one solution, $x = 1, y = 1$.

What happens to the two lines as you do row operations?

$$\left(\begin{array}{rr|r} 1 & 1 & 2 \\ -2 & 1 & -1 \end{array} \right) \rightsquigarrow$$

They pivot around the solution!