Math 1553 Supplement, §5.6 and §6.1

1. Suppose p and q are real numbers on the open interval $(0,1)$, and

$$A = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}$$

(1) Is A stochastic? Why?
(2) Does A have unique steady state vector? Why?
(3) By inspection(without computation), give an eigenvalue of A.
(4) Compute the steady-state vector of A.
(5) Compute the limit

$$\lim_{n\to\infty} A^n$$

2. $y = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$, $u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $(u)_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(1) Determine whether u_1 and u_2
   (a) are linearly independent
   (b) are mutually orthogonal
   (c) are orthonormal
   (d) span $\mathbf{R}^3$
(2) Is y in $W = \text{Span}\{u_1, u_2\}$?
(3) Compute the vector, $\widehat{y} \in W$, that most closely approximates y. [You may need orthogonal projections from §6.2]
(4) Construct a vector, z, that is in $W^{\perp}$.
(5) Make a rough sketch(use online tools) of $u_1, u_2, y, \widehat{y}$, and z.