Math 1553 Worksheet §5.4, 5.5

1. Answer yes, no, or maybe. Justify your answers. In each case, A is a matrix whose entries are real numbers.

a) If A is a 3×3 matrix with characteristic polynomial $-\lambda(\lambda - 5)^2$, then the 5-eigenspace is 2-dimensional.

b) If A is an invertible 2×2 matrix, then A is diagonalizable.

c) A 3×3 matrix A can have a non-real complex eigenvalue with multiplicity 2.

2. $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$.

a) Find the eigenvalues of A, and find a basis for each eigenspace.

b) Is A diagonalizable? If your answer is yes, find a diagonal matrix D and an invertible matrix C so that $A = CDC^{-1}$. If your answer is no, justify why A is not diagonalizable.