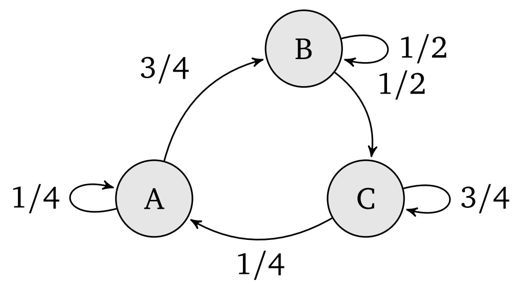
Math 1553 Worksheet §5.6 and §6.1

Solutions

1. There are three webpages linked in the following graph. For example, all people viewing page A will have 1/4 still viewing page A in the next hour, 3/4 switched to page B, etc.

(1) Identify the transition matrix and then compute the steady state vector. In the long run which webpage will have the highest ranking.



Solution.

(1)

$$P = \begin{pmatrix} 1/4 & 0 & 1/4 \\ 3/4 & 1/2 & 0 \\ 0 & 1/2 & 3/4 \end{pmatrix}$$

Then compute $Pq = q$ by $(P - I)q = 0$. Must make sure you select a probability vector from the solution set

(2) The steady state vector gives the long run behavior of the system. The third entry is the largest which means webpage C is the most popular web.

2. True/False

(1) If u is in subspace W, and u is also in $W^{\perp}$, then $u = 0$.

(2) If y is in subspace W, the orthogonal projection of y onto W is y.

(3) If x is orthogonal to v and w, then x is also orthogonal to $v - w$.

Solution.

(1) TRUE: $u \cdot u = ||u||^{1/2} = 0$ only when $u = 0$.
(2) TRUE: y is decomposed into the elements that form a basis for W, so they could be used to give a unique representation for y.
(3) TRUE: $v - w$ is on the same plane spanned by v and w.

3. Give examples

(1) two linearly independent vectors that are orthogonal to $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$.

(2) a subspace of $\mathbf{R}^3$, S, such that $\dim(S^{\perp}) = 2$.

Solution.

(1) Two linearly independent vectors orthogonal to the first vector can be found, for example, by setting

$$u = \begin{pmatrix} 1 \\ 0 \\ a \end{pmatrix}, v = \begin{pmatrix} 0 \\ 1 \\ b \end{pmatrix}$$

Taking dot products and solving for a and b gives the vectors we need. Students will likely get the answer by inspection in a way that takes advantage of the second element being zero, but you can ask how we might approach a more general problem.

(2) A few examples:

- The column space of $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
- The span of any vector in $\mathbb{R}^3$.
- The null space of a 3×3 matrix that has two pivots.