Math 1553 Worksheet §§3.4, 3.5, 3.6

1. If A is a 3×5 matrix and B is a 3×2 matrix, which of the following are defined?

a) $A - B$

b) AB

c) $A^T B$

d) $B^T A$

e) A^2

2. Consider the following linear transformations:

$$\begin{array}{ll} T : \mathbf{R}^3 \longrightarrow \mathbf{R}^2 & T \text{ projects onto the } xy\text{-plane, forgetting the } z\text{-coordinate} \\ U : \mathbf{R}^2 \longrightarrow \mathbf{R}^2 & U \text{ rotates clockwise by } 90^\circ \\ V : \mathbf{R}^2 \longrightarrow \mathbf{R}^2 & V \text{ scales the } x\text{-direction by a factor of 2.} \end{array}$$

Let A, B, C be the matrices for T, U, V, respectively.

a) Compute A, B, and C.

b) Compute the matrix for $V \circ U \circ T$.

c) Compute the matrix for $U \circ V \circ T$.

d) Describe U^{-1} and V^{-1}, and compute their matrices.

3. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then each column of AB is a linear combination of the columns of A.

b) If A and B are $n \times n$ and both are invertible, then the inverse of AB is $A^{-1}B^{-1}$.

c) If A^T is not invertible, then A is not invertible.

d) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

e) If A and B are invertible $n \times n$ matrices, then $A+B$ is invertible and $(A+B)^{-1} = A^{-1} + B^{-1}$.

f) If A and B are $n \times n$ matrices and $ABx = 0$ has a unique solution, then $Ax = 0$ has a unique solution.

4. Suppose A is an invertible 3×3 matrix with the following equations hold. Find A.

$$A^{-1}e_1 = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}, \quad A^{-1}e_2 = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \quad A^{-1}e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$