# Announcements Aug 25

- Please turn on your camera if you are able and comfortable doing so
- Mathematical autobiography due on Friday on Piazza (tag: margalit-autbio): picture, name/pronunciation, major, hobbies, math you've taken, feelings about math and/or class, plus anything else!
- Use Piazza for general questions
- WeBWorK Warmup due Fri (not for a grade). WeBWorK 1.1 due Tuesday
- Office hours for this week only: Thu 1-2 Skiles courtyard
- TA Office Hours
  - Ian tba
  - Patrick tba
  - Joseph tba
  - Ivan tba
  - Jieun tba
- Studio on Friday in person; Studio for M02 will be recorded/streamed
- Section M web site: Google me, click on Teaching, Math 1553
  - future blank slides, past lecture slides
- Tutoring: http://tutoring.gatech.edu/tutoring
- PLUS sessions: http://tutoring.gatech.edu/plus-sessions
- Math Lab: http://tutoring.gatech.edu/drop-tutoring-help-desks
- Counseling center: https://counseling.gatech.edu

Canvas →
Assignments
→ Webwork

# Applications of Linear Algebra

**Civil Engineering:** How much traffic lies in the four unlabeled segments?
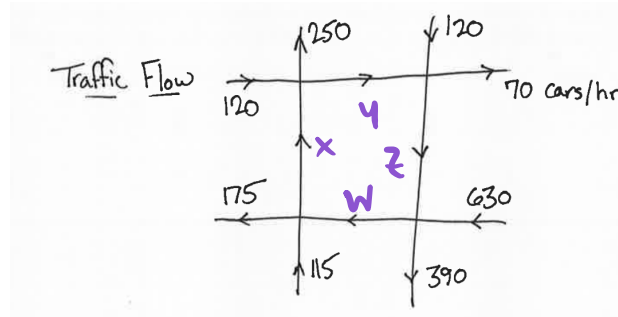


For each intersection

$$\# \text{ cars in} = \# \text{ cars out}$$

$$\rightsquigarrow \quad x + 120 = y + 250$$

3 other eqns...

# Applications of Linear Algebra

**Biology:** In a population of rabbits...

- half of the new born rabbits survive their first year
- of those, half survive their second year
- the maximum life span is three years
- *each* rabbits produce 0, 6, 8 rabbits in their first, second, and third years

If the numbers of first, second, and third year rabbits in 2021 are $10$, $4$, and $5$, then what are they in 2022?

$$F_{2022} = 6 \cdot S_{2021} + 8 \cdot T_{2021} = 6 \cdot 4 + 8 \cdot 5$$
$$S_{2022} = \tfrac{1}{2} F_{2021} \qquad\qquad = \tfrac{1}{2} \cdot 10$$
$$T_{2022} = \tfrac{1}{2} S_{2021} \qquad\qquad = \tfrac{1}{2} \cdot 4$$

If the numbers of first, second, and third year rabbits in year $n$ are $F_n$, $S_n$, and $T_n$, what are the numbers in year $n+1$?

$$F_{n+1} = 6 \cdot S_n + 8 \cdot T_n$$
$$S_{n+1} = \tfrac{1}{2} \cdot F_n$$
$$T_{n+1} = \tfrac{1}{2} \cdot S_n$$

What happens in the long term?

▶ Demo

Magical thing: population ratios $\sim 16 : 4 : 1$ & population $\sim$ doubles each year

# Section 1.1

Solving systems of equations

# Outline of Section 1.1

- Learn what it means to solve a system of linear equations
- Describe the solutions as points in $\mathbb{R}^n$
- Learn what it means for a system of linear equations to be inconsistent

# Solving equations

# Solving equations

What does it mean to solve an equation?

$2x = 10$      $x = 5$

$x + y = 1$

$x + y + z = 0$      one soln: $(0,0,0) \rightsquigarrow x = 0, y = 0, z = 0.$
$(1,1,-2)$
$(2,-1,-1)$

Find one solution to each. Can you find all of them?

$\hookrightarrow$ yes!

A solution is a *list* of numbers (a.k.a. a *vector*). For example $(3, -4, 1)$.

# Solving equations

What does it mean to solve a system of equations?

$$x + y = 2$$
$$y = 1$$

*Finding x, y that work for both eqns.*

*x = 1*
*y = 1*

What about...

$$x + y + z = 3$$
$$x + y - z = 1$$
$$x - y + z = 1$$

*Harder...*

Is $(1, 1, 1)$ a solution? Is $(2, 0, 1)$ a solution? What are all the solutions?
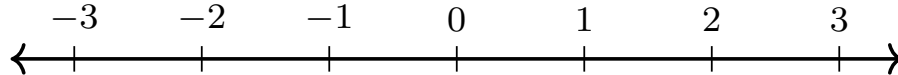
*✗ fails*
*3rd eqn*

Soon, you will be able to see just by looking that there is exactly one solution.

$$\mathbb{R}^n$$

$\mathbb{R} =$ denotes the set of all real numbers

Geometrically, this is the *number line*.



A point in $\mathbb{R}^4$:

$(0,0,0,0)$

$(1,2,3,4)$

$(\pi, e, -1, \sqrt{3})$

$\mathbb{R}^n =$ all ordered $n$-tuples (or lists) of real numbers $(x_1, x_2, x_3, \ldots, x_n)$

Solutions to systems of equations are exactly points in $\mathbb{R}^n$. In other words, $\mathbb{R}^n$ is where our solutions will lie (the $n$ depends on the system of equations).

We say $\mathbb{R}^n$ instead of $\mathbb{R}^2$ or $\mathbb{R}^3$ because many of the things we learn this semester work just as well for $\mathbb{R}^{777}$ as they do for $\mathbb{R}^2$ and $\mathbb{R}^3$. So when we say $\mathbb{R}^n$ we are talking about all of these at once. That is power!
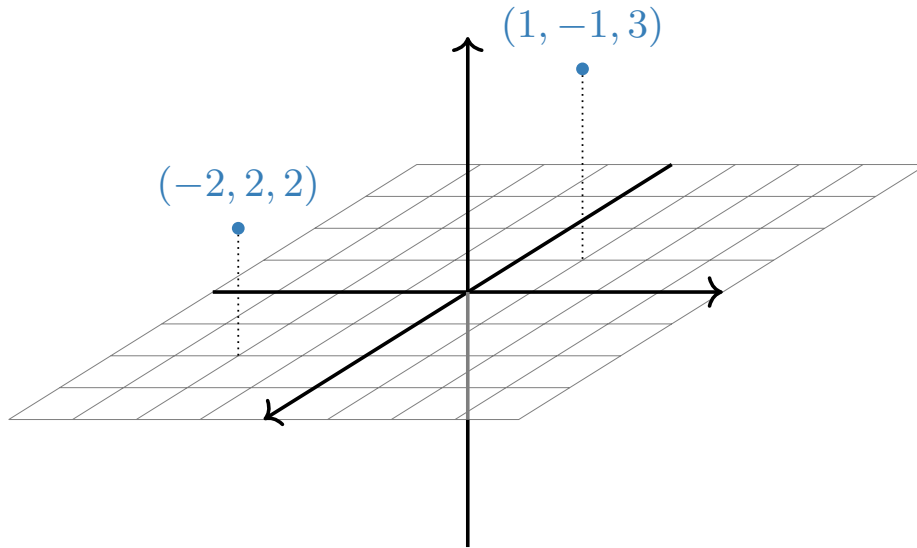
$\mathbb{R}^n$

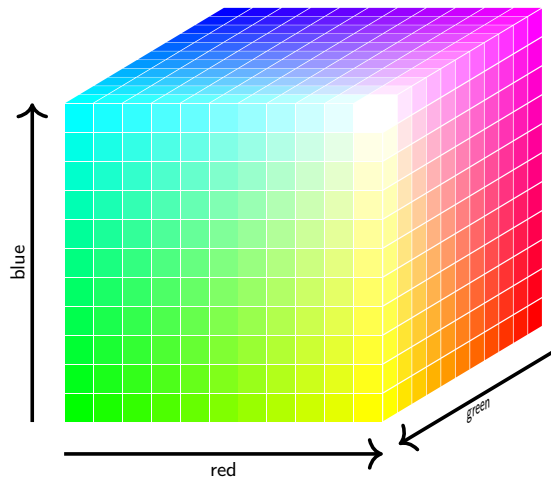When $n = 2$, we can visualize of $\mathbb{R}^2$ as the *plane*.
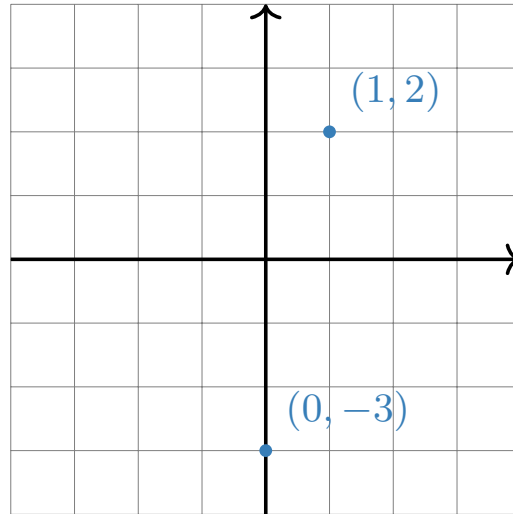
When $n = 3$, we can visualize $\mathbb{R}^3$ as the *space* we (appear to) live in.

$(1, -1, 3)$

$(-2, 2, 2)$

$\mathbb{R}^3$ is 3D space

a point in $\mathbb{R}^3$ is a list of 3 numbers

$\mathbb{R}^n$

We can think of the space of all *colors* as (a subset of) $\mathbb{R}^3$:

So what is $\mathbb{R}^4$? or $\mathbb{R}^5$? or $\mathbb{R}^n$?

. . . go back to the *definition*: ordered $n$-tuples of real numbers

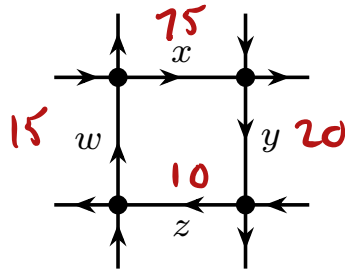$$(x_1, x_2, x_3, \ldots, x_n).$$

They're still "geometric" spaces, in the sense that our intuition for $\mathbb{R}^2$ and $\mathbb{R}^3$ sometimes extends to $\mathbb{R}^n$, but they're harder to visualize.

Last time we could have used $\mathbb{R}^3$ to describe a rabbit population in a given year: (first year, second year, third year).

$$(5, 3, 7)$$

Similarly, we could have used $\mathbb{R}^4$ to label the amount of traffic $(x, y, z, w)$ passing through four streets.
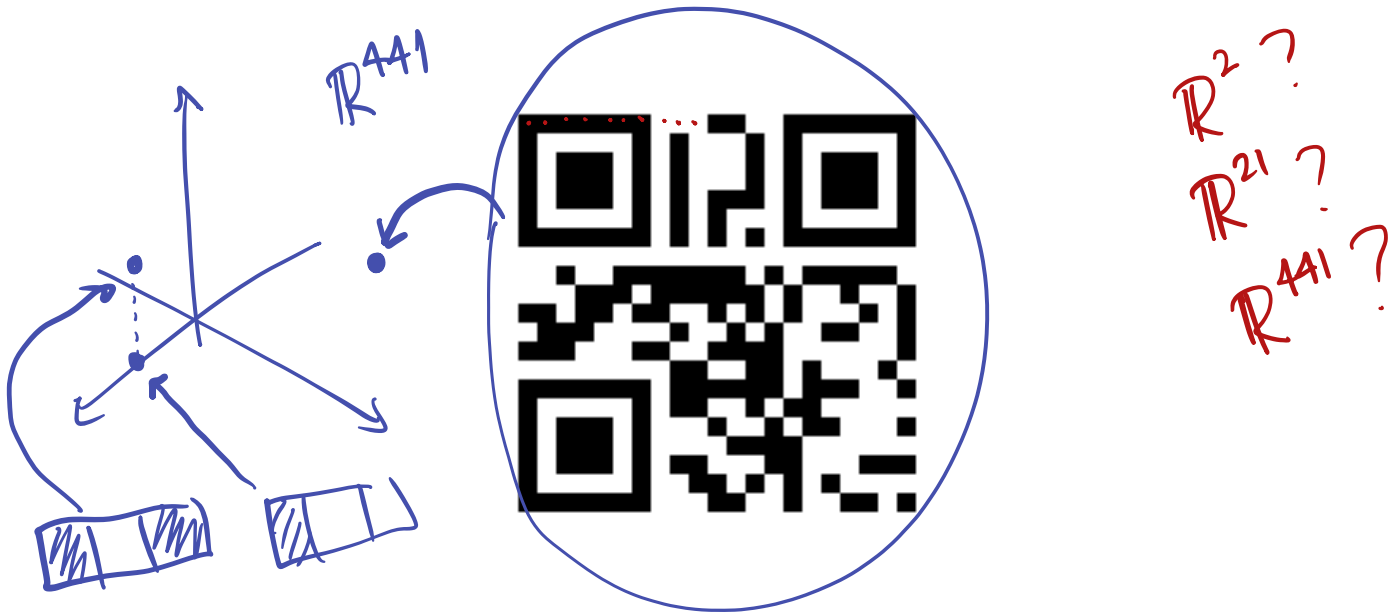


$$(75, 20, 10, 15)$$

We'll make definitions and state theorems that apply to any $\mathbb{R}^n$, but we'll only draw pictures in $\mathbb{R}^2$ and $\mathbb{R}^3$.

$\mathbb{R}^n$

This is a $21 \times 21$ QR code. We can also think of this as an element of $\mathbb{R}^n$.



$\mathbb{R}^{441}$

$\mathbb{R}^2$ ?

$\mathbb{R}^{21}$ ?

$\mathbb{R}^{441}$ ?

How? Which $n$?  $\mathbb{R}^{441}$  $\left(1,1,1,1,1,1,1,0,0,0,\ldots\right)$
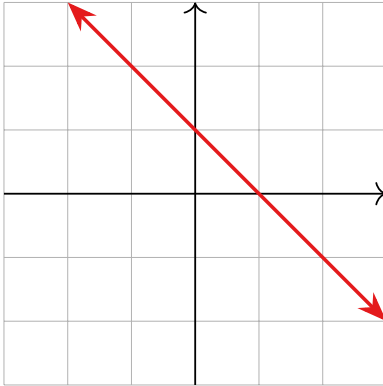
What about a greyscale image?

This is a powerful idea: instead of thinking of a QR code as 441 pieces of information, we think of it as one piece of information.

# Visualizing solutions: a preview

# One Linear Equation

What does the solution set of a linear equation look like?
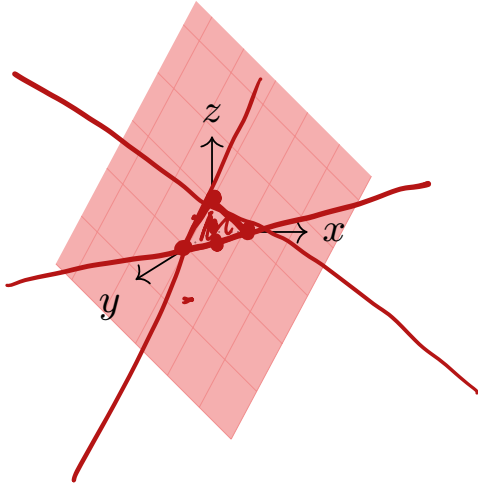
$x + y = 1$ ⤳ a line in the plane: $y = 1 - x$

Algebra
↕
Geometry

# One Linear Equation

What does the solution set of a linear equation look like?

$x + y + z = 1$ $\rightsquigarrow$ a plane in space:



$$x + y = 1$$
$$z = 0.$$

# One Linear Equation

What does the solution set of a linear equation look like?

$x + y + z + w = 1$ ⤳ a "3-plane" in "4-space"...

3D plane     $\mathbb{R}^4$

1 eqn in 100 vars

Solns: 99-dim plane in $\mathbb{R}^{100}$

# Systems of Linear Equations

What does the solution set of a *system* of more than one linear equation look like?

$$x - 3y = -3$$
$$2x + y = 8$$

Intersection of 2 lines in $\mathbb{R}^2$

In this case:
1 pt.
(lines not same/parallel)

What are the other possibilities for two equations with two variables?

Soln can be: 1 pt, $\infty$ solns, no solns $\longrightarrow$ $x = 0$
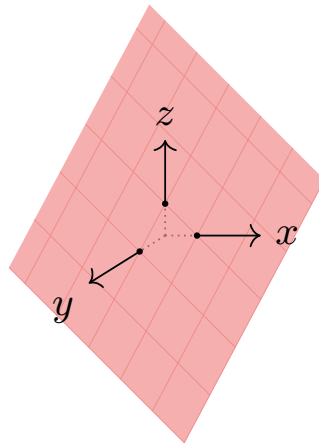$x = 1$

What if there are more variables? More equations? $\searrow$ $x + y = 1$
$2x + 2y = 2$

Take this class!

Is the plane $x + y + z = 1$ in $\mathbb{R}^3$ equal to $\mathbb{R}^2$? What about the $xy$-plane in $\mathbb{R}^3$?

1. yes + yes
2. yes + no
3. no + yes
4. no + no

# Consistent versus Inconsistent
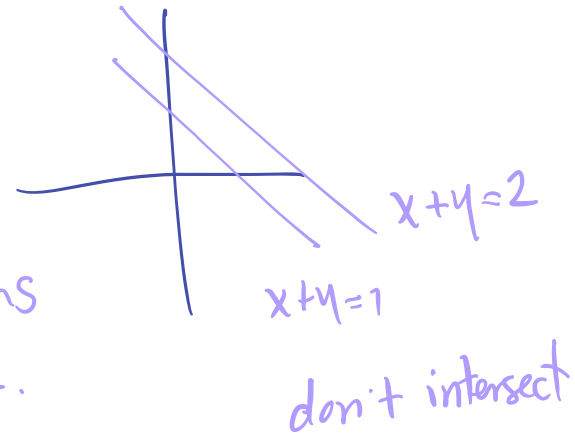
We say that a system of linear equations is **consistent** if it has a solution and **inconsistent** otherwise.

$$x + y = 1$$
$$x + y = 2$$

Why is this inconsistent?
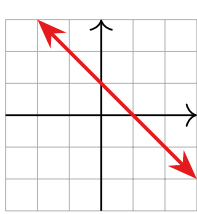
Can't have a pair of nums
that sum to 1 & 2.

No point $(x,y)$ in $\mathbb{R}^2$
satisfies both eqns.

$x+y=2$

$x+y=1$

don't intersect

What are other examples of inconsistent systems of linear equations?

e.g.     $x + y + z = 1$  (same as $2x + 2y + 2z = 2$)
$2x + 2y + 2z = 1$

# Parametric form

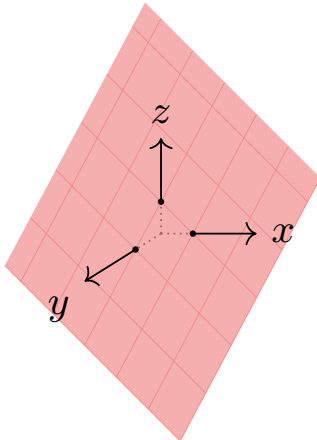The equation $2x + 2y = 2$ is an implicit equation for the line in the picture.

$x + y = 1$

It also has a parametric form: $(x, 1-x)$.

put in anything for x — to get all solns. — $(e, 1-e)$ $(7, 1-7)$

The difference is that in the parametric form you get to plug in whatever you want for all variables. There's no guesswork, and no solving of anything.

Similarly the equation $x + y + z = 1$ is an implicit equation. One parametric form is: $(x, y, 1 - x - y)$.

$x = 1, y = 0 \rightsquigarrow (1, 0, 0)$

$x = 5, y = 7 \rightsquigarrow (5, 7, -11)$

# Parametric form

The equation $y = 1 - x$ is an implicit equation for the line in the picture.



It also has a parametric form: $(x, 1 - x)$.

The difference is that in the parametric form you get to plug in whatever you want for all variables. There's no guesswork, and no solving of anything.

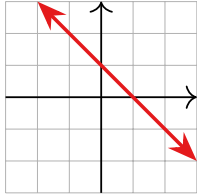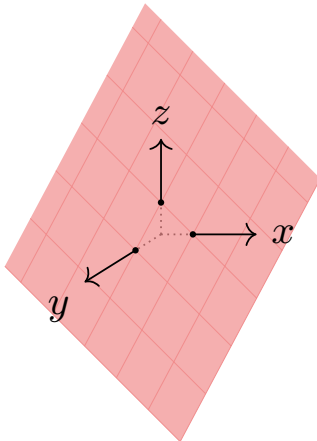Similarly the equation $x + y + z = 1$ is an implicit equation. One parametric form is: $(x, y, 1 - x - y)$.



What is an implicit equation and a parametric form for the $xy$-plane in $\mathbb{R}^3$?

# Parametric form
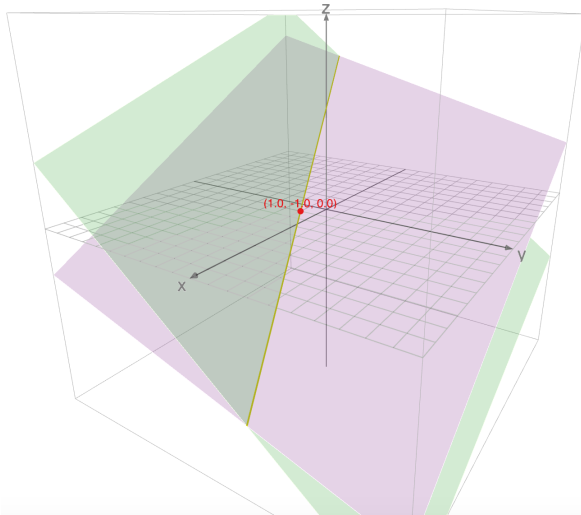
The system of equations

$$2x + y + 12z = 1$$
$$x + 2y + 9z = -1$$

is the implicit form for the line of intersection in the picture.

↪ unsolved system of lin eqns.



$z = 0 : (1, -1, 0)$
$z = 5 : (-24, -11, 5)$

explicit

The line of intersection also has a parametric form: $(1 - 5z, -1 - 2z, z)$

We think of the former as being the problem and the latter as being the explicit solution. One of our first tasks this semester is to learn how to go from the implicit form to the parametric form.

# Summary of Section 1.1

- A solution to a system of linear equations in $n$ variables is a point in $\mathbb{R}^n$.

- The set of all solutions to a single equation in $n$ variables is an $(n-1)$-dimensional plane in $\mathbb{R}^n$

- The set of solutions to a system of $m$ linear equations in $n$ variables is the intersection of $m$ of these $(n-1)$-dimensional planes in $\mathbb{R}^n$.

- A system of equations with no solutions is said to be inconsistent.

- Line and planes have implicit equations and parametric forms.

# Typical exam questions

Write down and example of a point in $\mathbb{R}^7$.

Find all values of $h$ so that the following system of linear equations is consistent:
$$x + y + z = 2$$
$$2x + 2y + 2z = h$$

True/False: Points in $\mathbb{R}^3$ are also points in $\mathbb{R}^4$.

Find two different parametric solutions to the equation $x - 3y = 5$.

True/False: the set of solutions to $x_1 = 1$ in $\mathbb{R}^5$ is a line.

# Section 1.2

Row reduction

# Outline of Section 1.2

- Solve systems of linear equations via elimination
- Solve systems of linear equations via matrices and row reduction
- Learn about row echelon form and reduced row echelon form of a matrix
- Learn the algorithm for finding the (reduced) row echelon form of a matrix
- Determine from the row echelon form of a matrix if the corresponding system of linear equations is consistent or not.

# Solving systems of linear equations by elimination

# Example

Solve:

$$-y + 8z = 10$$
$$5y + 10z = 0$$

How many ways can you do it?

# Example

Solve:

$$-x + y + 3z = -2$$
$$2x - 3y + 2z = 14$$
$$3x + 2y + z = 6$$

Hint: Eliminate $x$!

# Solving systems of linear equations with matrices

# Example

Solve:

$$-y + 8z = 10$$
$$5y + 10z = 0$$

It is redundant to write $x$ and $y$ again and again, so we rewrite using (augmented) *matrices*. In other words, just keep track of the coefficients, drop the $+$ and $=$ signs. We put a vertical line where the equals sign is.

$$\left( \begin{array}{cc|c} -1 & 8 & 10 \\ 5 & 10 & 0 \end{array} \right) \rightsquigarrow$$